EVALUATION OF MODELS TO PREDICT INSOLATION ON TILTED SURFACES

Thomas M. Klucher
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

March 1978

Prepared for
U.S. DEPARTMENT OF ENERGY
Division of Solar Energy
Under Interagency Agreement E(49-26)-1022
NOTICE

This report was prepared to document work sponsored by the United States Government. Neither the United States nor its agent, the United States Department of Energy, nor any Federal employees, nor any of their contractors, subcontractors or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.
An empirical study was performed to evaluate the validity of various insolation models which employ either an isotropic or an anisotropic distribution approximation for sky light when predicting insolation on tilted surfaces. Data sets of measured hourly insolation values were obtained over a six-month period using pyranometers which received diffuse and total solar radiation on a horizontal plane and total radiation on surfaces tilted toward the equator at 37° and 60° angles above the horizon. Data on the horizontal surfaces were used in the insolation models to predict insolation on the tilted surface; comparisons of measured versus calculated insolation on the tilted surface were examined to test the validity of the sky light approximations. It was found that the Liu-Jordan isotropic distribution model provides a good fit to empirical data under overcast skies but underestimates the amount of solar radiation incident on tilted surface under clear and partly cloudy conditions. The anisotropic-clear-sky distribution model by Temps and Coulson provides a good prediction for clear skies but overestimates the solar radiation when used for cloudy days. An anisotropic-all-sky model was formulated in this effort which provided excellent agreement between measured and predicted insolation throughout the six-month period.
EVALUATION OF MODELS TO PREDICT INSOLATION
ON TILTED SURFACES

by Thomas M. Klucher

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

An empirical study was performed to evaluate the validity of various insolation models which employ either an isotropic or an anisotropic distribution approximation for sky light when predicting insolation on tilted surfaces. Data sets of measured hourly insolation values were obtained over a six-month period using pyranometers which received diffuse and total solar radiation on a horizontal plane and total radiation on surfaces tilted toward the equator at 37° and 60° angles above the horizon. Data on the horizontal surfaces were used in the insolation models to predict insolation on the tilted surface; comparisons of measured versus calculated insolation on the tilted surface were examined to test the validity of the sky light approximations. It was found that the Liu-Jordan isotropic distribution model provides a good fit to empirical data under overcast skies but underestimates the amount of solar radiation incident on tilted surface under clear and partly cloudy conditions. The anisotropic-clear-sky distribution model by Temps and Coulson provides a good prediction for clear skies but overestimates the solar radiation when used for cloudy days. An anisotropic-all-sky model was formulated in this effort which provided excellent agreement between measured and predicted insolation throughout the six-month period.

INTRODUCTION

Flat plate solar heating/cooling or solar electric arrays in field applications are usually oriented at a fixed tilt angle to maximize the solar radiation falling on the array for the period of use. Occasionally, angle adjustments are provided to optimize the insolation for shorter periods (i.e., monthly or seasonally). Accurate prediction of the tilt angle for best array performance is complicated, however, by insufficient knowledge of the exact magnitude of solar radiation falling on
tilted surfaces. Estimates of insolation for a given array orientation and location usually cannot be based on radiation data on tilted surfaces because such data are generally not available. Instead, the historical data bases of climatic variables, which include cloud cover, percentage of sunshine, and solar radiation on horizontal surfaces, are generally used in combination with some insolation model to estimate insolation on tilted surfaces. Selection of the tilt angle is then dependent on the procedures and approximations employed in the insolation model.

A number of insolation models have been developed which predict the amount of solar radiation incident on tilted arrays from historical data of solar radiation falling on a horizontal surface. Because of the variety and complexity of sky radiation, a common assumption made is that the diffuse component of solar radiation (sky light) has an isotropic distribution (refs. 1 and 2) over the hemispherical sky. This assumption conveniently allows the direct and diffuse components on the horizontal plane to be corrected to total radiation on the inclined plane by simple geometric relationships. However, it has been shown that sky light is anisotropic in many instances and that the assumption of an isotropic distribution may introduce significant error in modelling calculations for tilted surfaces (refs. 3 and 4). Recently, a model for improving the diffuse radiation approximation for clear skies was introduced by Temps and Coulson (ref. 5); simple correction terms were developed to account for the anisotropy of the diffuse radiation field. Although the model gave good results for clear sky conditions, it was not extended to the cases for partly cloudy and overcast skies.

The objective of the present effort (a part of the National Photovoltaic Conversion Program established by the Department of Energy) was to test the validity of certain insolation models which use either an isotropic or anisotropic sky light distribution approximation to predict insolation on tilted surfaces under cloudy as well as clear sky conditions. The approach taken to achieve this objective was to establish a data set over a six-month period (January-June 1977) in Cleveland, Ohio, consisting of hourly average insolation measured by pyranometers on horizontal and inclined planes. Total and diffuse radiation data in the horizontal plane were utilized in the Liu-Jordan insolation model (ref. 2) to predict insolation falling on tilted surfaces under the assumption of isotropic sky light. The isotropic model was then modified by the Temps-Coulson clear-sky (ref. 5) correction terms to evaluate that anisotropic model under all sky conditions. Finally, the anisotropic-clear-sky model was modified when it was found inadequate to account for cloudy sky conditions.
Evaluation of the validity of the three models was made by comparing the solar radiation measured on the tilted surfaces with the radiation predicted by each insolation model.

APPARATUS AND MEASUREMENTS

The test facility used to measure, catalog, and process the hourly solar irradiance data has been previously described in reference 6. Figure 1 shows the details of sensor orientation. Three precision pyranometers measure total insolation received at 0°, 37°, and 60° tilt angles facing due south. The diffuse component is measured by a fourth pyranometer horizontally mounted (0° tilt) and equipped with a black shadow band to block out the direct radiation component. The amount of diffuse radiation which is screened by the shadow band was added to the measured values by application of a correction factor recommended by Drummond (ref. 7). The pyranometers tilted at 37° and 60° were equipped with black metal artificial horizon bands, which prevented any radiation reflected from below the horizon from reaching the sensors.

Each pyranometer was corrected to a voltage-controlled oscillator and a counter which integrated the data continuously. The integrated data were monitored every hour and time averages of the hourly data were obtained for evaluation.

METHOD OF ANALYSIS

The starting point for the evaluation of insolation models was the isotropic sky model described by Liu and Jordan in reference 2. In this model, the insolation on a surface tilted toward the equator at an angle $\epsilon$ to the horizontal is given by:

$$I_T = \left( \frac{I_H - I_D}{\sin \alpha} \cos \psi + I_D \left( \frac{1 + \cos \epsilon}{2} \right) \right)$$

$$=$$

Direct radiation + Diffuse radiation

(1)
where

\[ I_T \] total insolation received by tilted surface
\[ I_H \] total insolation received by horizontal surface
\[ I_D \] diffuse insolation received by horizontal surface
\[ \alpha \] solar elevation angle
\[ \psi \] angle between sun direction and normal direction
\[ \varepsilon \] tilt angle above horizon of tilted surface

In this effort the insolation terms inserted into equation (1) were hourly average values of insolation obtained from sunrise to sunset during each day. The geometric terms, \( \cos \psi \) and \( \sin \alpha \), were hourly average values calculated from well established geophysical and astronomical equations (refs. 2 and 8). The insolation \( I_H \) and \( I_D \) were measured by the pyranometers and used in equation (1) to calculate the total insolation received on the surfaces tilted at 37° and 60°. These calculated insolation values were then compared with the insolation \( I_T \) measured at 37° and 60° to determine how well the Liu-Jordan model predicted the insolation on each tilted surface.

The data were also compared with the anisotropic-clear-sky model developed by Temps and Coulson. In their model Temps and Coulson combined three correction factors with the isotropic diffuse radiation term to account for each of three regions of anisotropy in the diffuse radiation field. They determined that a factor, \( 1 + \sin^3(\varepsilon/2) \), accounts for the increase in sky light observed near the horizon during clear days; similarly, sky brightening near the sun could be approximated by the factor \( 1 + \cos^2 \psi \sin^3(90 - \alpha) \). A third factor which accounts for surface reflection enhancement is not included in this report because this effect was eliminated by the horizon shades previously described. Applying the Temps and Coulson correction terms to the Liu-Jordan model, then, the anisotropic-clear-sky model has the form:

\[
I_T = \frac{(I_H - I_D)}{\sin \alpha} \cos \psi + I_D \left( \frac{1 + \cos \varepsilon}{2} \right) \left[ 1 + \sin^3 \frac{\varepsilon}{2} \right] \left[ 1 + \cos^2 \psi \sin^3 (90 - \alpha) \right]
\] (2)

Here again, measured values for radiation on tilted surfaces were compared to those calculated from measured values of total and diffuse radiation on horizontal surfaces to determine the fit of the model.
The final model evaluated was an anisotropic model developed in this effort based upon preliminary results found with the previous two models. This last model involves an adjustment to the Temps-Coulson factors by a simple function containing the ratio of diffuse to total insolation on the horizontal plane. As will be shown in the RESULTS AND DISCUSSION, the Liu-Jordan model worked well for overcast days and the Temps-Coulson model worked well for clear days. The purpose of the new function was to modulate the Temps-Coulson factors as the skies varied from clear to overcast. This anisotropic, "all sky" model thus takes the form:

\[
I_T = \frac{(I_H - I_D)}{\sin \alpha} \cos \psi + I_D \left(1 + \frac{1 + \cos \epsilon \cos \psi}{2}ight) \left[1 + F \sin^3 \frac{\epsilon}{2} \sin^3 (90 - \alpha)\right] (3)
\]

where \( F \) is the modulating function described above. Two slightly different functions, \( F = 1 - I_D/I_H \), and \( F = 1 - (I_D/I_H)^2 \), were studied in the all-sky model. Under overcast conditions, when the ratio of diffuse to total insolation, \( I_D/I_H \), is unity, the all-sky model, using either "F", reduces to the Liu-Jordan isotropic model. Under clear sky, when the ratio of diffuse to total is observed to be small, the all-sky model approximates the Temps-Coulson anisotropic-clear-sky model.

RESULTS AND DISCUSSION

Comparison plots of hourly measured versus calculated solar irradiance at 37° and 60° tilt angles were made for each month and each model to determine how well each model predicts insolation on a tilted surface. A total of 48 plots were examined for the six-month period from January to June 1977. In the interest of clarity, selected plots of the monthly data were chosen from this group for discussion of the major results of this effort. A complete set of monthly plots for the three models are shown for comparative purposes at the end of the report (figs. 16 to 27).

Figures 2 to 7 illustrate the results of the application of the Liu-Jordan model to the prediction of solar radiation falling on the tilted sensor for the months of January, March, and June for 37° and 60°. Inspection of each plot indicates that the model provides a good fit to the empirical data at the low intensity conditions (<20 to 30 mw/cm²); this might be expected, since the low intensities are primar-
ily associated with the occurrence of overcast sky conditions, little direct insolation, and uniform diffuse sky radiation. In addition, each of the plots also demonstrate a nearly linear correlation between computed and measured insolation. At the higher intensities (>50 mw/cm\(^2\)), however, figures 2 to 7 illustrate that the Liu-Jordan model underestimates the amount of solar radiation falling on tilted surfaces. This error is greatest in January — about 5 mw/cm\(^2\) at 37° and 8 mw/cm\(^2\) at 60° tilt angles — and decreases to about 3 mw/cm\(^2\) in June. These results clearly demonstrate that the isotropic sky model is deficient in predicting insolation on tilted surfaces for nonuniform (clear and partly cloudy) sky conditions.

Figures 8 and 9 illustrate typical results obtained when comparisons of measured and calculated insulations are made using the anisotropic-clear-sky model of Temps and Coulson under partly cloudy and overcast, as well as clear-sky conditions. Data for days on which the sky was observed and recorded as clear and basically cloudless were inspected as an initial check of the Temps-Coulson regression model for clear days. The comparison for clear days in March (figs. 8 and 9) were very good; the hourly data were found to fall along the 1:1 correlation line in the figures, indicating a good fit between measured and predicted clear-sky insolation. Such good comparisons for clear-sky data were found to be true throughout the six-month period of this effort and confirm the results of Temps and Coulson. On the other hand, during nonclear days, the comparison plots demonstrate that application of the Temps-Coulson corrective terms to overcast and cloudy conditions often result in an overestimate of the solar radiation falling on a tilted surface. Figures 8 and 9 show predicted values exceeding measured by as much as 10 mw/cm\(^2\); such overestimates are greater than 10 mw/cm\(^2\) in winter and smallest in summer. In their report, Temps and Coulson speculated that their sky model might improve modeling predictions for scattered cloud conditions, but would not give good results for heavily clouded and overcast skies. The results in this six-month study are consistent with their expectations.

Comparison plots of the anisotropic-all-sky insolation model are shown in figures 10 to 15. The model using parameter \( F = 1 - (I_D/I_H)^2 \) in equation (3) was chosen because it provides a slightly better fit to the empirical data than does the linear parameter \( F = 1 - I_D/I_H \).

The primary effect of correction terms in the all-sky model was a reduction in the systematic error previously observed in the January and March plots of the Liu-Jordan model. For example, the systematic error in January is reduced to
less than 2.5 mw/cm$^2$; similar reductions also occur in the March plots. In June, when the Liu–Jordan model has a good fit to the measured data, the two models appear to fit equally well. Comparisons between the Temps–Coulson clear-sky model and the all-sky model also demonstrate the effect of the correction terms used in the all-sky model. While the error in January to June plots are less than 2.5 mw/cm$^2$ in the all-sky model, the errors in the Temps–Coulson model results range from about 12 mw/cm$^2$ in January to 3 to 5 mw/cm$^2$ in June during nonclear days. Since the accuracy of first-class pyranometer measurements is, at best, ±2 mw/cm$^2$, the all-sky model is considered to fit well for the six months studied. Thus the anisotropic-all-sky model formulated in this effort provides a better prediction of solar radiation on tilted surfaces than either the isotropic or anisotropic-clear-sky models.

CONCLUSIONS

As a result of the study of three insolation models which predict the solar radiation incident on tilted surfaces from measured values of total and diffuse radiation on a horizontal surface, the following conclusions were formed:

1. The commonly used Liu–Jordan isotropic-sky insolation model provides a good fit to empirical data at low intensity conditions found during overcast skies; however, the model underestimates the amount of solar radiation falling on tilted surfaces at intensity levels above about 5 mw/cm$^2$. In this effort the error is about 3 mw/cm$^2$ in June and rises to 5 to 8 mw/cm$^2$ in January.

2. The anisotropic-clear-sky model of Temps and Coulson provides a good correlation between measured and predicted insolation on tilted surfaces for the case of clear skies. However, this model overestimates the insolation for mostly cloudy and overcast conditions by as much as 12 mw/cm$^2$ in January and from 3 to 5 mw/cm$^2$ in June.

3. The anisotropic-all-sky model formulated in this effort provides a better prediction of solar radiation on tilted surfaces than either the isotropic or anisotropic-clear-sky models. The systematic error between measured and predicted insolation resulting from the application of the anisotropic-all-sky model is less than 2.5 mw/cm$^2$. 

REFERENCES


Figure 1. - Sensor orientation.
Figure 2. - Comparison between measured and calculated insolation on a tilted surface using the Liu-Jordon isotropic-sky model.

Figure 3. - Comparison between measured and calculated insolation on a tilted surface using the Liu-Jordon isotropic-sky model.
Figure 4. - Comparison between measured and calculated insolation on a tilted surface using the Liu-Jordon isotropic-sky model.

Figure 5. - Comparison between measured and calculated insolation on a tilted surface using the Liu-Jordon isotropic-sky model.
Figure 6. - Comparison between measured and calculated insolation on a tilted surface using the Liu-Jordan isotropic-sky model.

Figure 7. - Comparison between measured and calculated insolation on a tilted surface using the Liu-Jordan isotropic-sky model.
Figure 8. - Comparison between measured and calculated insolation on a tilted surface using the Temps-Coulson anisotropic-clear-sky method.

Figure 9. - Comparison between measured and calculated insolation on a tilted surface using the Temps-Coulson anisotropic-clear-sky method.
Figure 10. - Comparison between measured and calculated insolation on a tilted surface using the anisotropic-all-sky model.

Figure 11. - Comparison between measured and calculated insolation on a tilted surface using the anisotropic-all-sky model.
Figure 12. - Comparison between measured and calculated insolation on a tilted surface using the anisotropic-all-sky model.

Figure 13. - Comparison between measured and calculated insolation on a tilted surface using the anisotropic-all-sky model.
Figure 14. Comparison between measured and calculated insolation on a tilted surface using the anisotropic-all-sky model.

Figure 15. Comparison between measured and calculated insolation on a tilted surface using the anisotropic-all-sky model.
Figure 16. Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 17. - Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 18. Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 15. - Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 20. Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 21. - Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 22. - Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 23. Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 24. Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 25. Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 26 - Comparison between measured and calculated insolation on a tilted surface for each sky model.
Figure 27: Comparison between measured and calculated insolation on a tilted surface for each sky model.