Flat-Plate Photovoltaic Module & Array Circuit Design Optimization
Workshop Proceedings
Engineering Area

May 19 & 20, 1980

Prepared for
U.S. Department of Energy
Through an agreement with
National Aeronautics and Space Administration
by
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California
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The JPL Low-Cost Solar Array Project is sponsored by the Department of Energy (DOE) and forms part of the Solar Photovoltaic Conversion Program to initiate a major effort toward the development of low-cost solar arrays.

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PHOTOVOLTAIC ARRAY CIRCUIT DESIGN WORKSHOP

Agenda

May 19

2:00  Registration and Coffee
2:30  Introduction

2:30

- Welcome/Meeting Objectives  Ross
- Agenda/Announcements/Handouts  Gonzalez

3:00  Background Review

3:00

- Nomenclature and I-V Characteristics  Ross
- Failure Probability Statistics  Weaver
- Module Design Constraints  Sugimura

5:00  Adjourn

May 20

7:30-8:00  Coffee Available
8:00  Mismatch Losses  Gonzalez/Cox
8:40  Manufacturing Yield  Gonzalez
9:20  Hot-Spot Heating Constraints  Ross
10:00  Coffee
10:15  Array Fault Tolerance  Gonzalez

10:15

- Field Failure Statistics
- Array Power Loss Calculations

11:30  Lunch
12:30  Overall Array Design Optimization  Weaver
1:15  Simplified Design Methodology  Ross
1:45  Working of Example Problem  Ross
2:15  Working of Example Problem by Participants

3:00  Coffee
3:30  Discussion of Example Problem  All

5:00  Adjourn

Engineering Area
Low-Cost Solar Array Project
Jet Propulsion Laboratory
Pasadena, California
May 19-20, 1980
FOREWORD

This document contains the proceedings of the Flat-Plate Photovoltaic Module and Array Circuit Design Optimization Workshop held at JPL during May of 1980. The workshop was held to discuss the problem of optimizing terrestrial photovoltaic array cell and circuit reliability and the methods of resolving this problem.

Certain faults are present in terrestrial photovoltaic modules, both at the beginning of life and throughout field experience. These faults occur as a result of mismatch of photovoltaic cell characteristics, both initial mismatch and mismatch caused by field environmental stresses and maintenance procedures. They include open-circuit cell interconnects, cracked cells and cell shading.

The objective of the workshop was to investigate the effectiveness of certain circuit-design strategies in ameliorating the effects of faults on module and array-system performance. A set of guidelines was presented for use in developing module and array-system circuit design strategies that maximize reliability through use of fault-tolerant circuiting. For simplicity, the open-circuit cell interconnect failure mode was emphasized in the workshop.

Cost and efficiency values presented in the proceedings are values accepted at the time of the workshop. All costs are in 1975 dollars.
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D. POWER LOSS IN PHOTOVOLTAIC ARRAYS DUE TO
   MISMATCH IN CELL CHARACTERISTICS ........... D-1


V
LOW-COST SOLAR ARRAY PROJECT
PHOTOVOLTAIC ARRAY NOMENCLATURE

ARRAY

SUPERSTRATE
INTERCONNECT
CELL
SUBSTRATE

TOP METALLIZATION (-)

N-TYPE
DIFFUSED LAYER

P-N JUNCTION

P-TYPE WAFER
BOTTOM METALLIZATION (+)
LOW-COST SOLAR ARRAY PROJECT

SERIES/PARALLEL NOMENCLATURE

MODULE:
- 3 PARALLEL STRINGS
- 2 SERIES BLOCKS
- 2 CELLS PER SUBSTRING
- 2 DIODES PER MODULE

BRANCH CIRCUIT:
- 3 PARALLEL STRINGS
- 6 SERIES BLOCKS
- 2 CELLS PER SUBSTRING
- 1 DIODE PER SERIES BLOCK
LOW-COST SOLAR ARRAY PROJECTS

TYPICAL SOLAR CELL I-V CURVE

SHUNT RESISTANCE = $\frac{\Delta V}{\Delta I}$

MAX

AVG

MIN

$\Delta I \approx \frac{I_{SC}}{10}$

MAX POWER

$V_{OC}$

$\Delta V$
LOW - COST SILICON SOLAR ARRAY PROJECT

CELL SHUNT RESISTANCE MEASUREMENT SETUP

$V_s = V_M \left( \frac{n-1}{n} \right) - V_{MS}$,

$R = \frac{V_s}{I}$
LOW - COST SILICON SOLAR ARRAY PROJECT

CELL SHUNT RESISTANCE

NUMBER OF CELLS PER RESISTANCE CATEGORY

CELL SHUNT RESISTANCE, ohms

1.0 1.5 2.2 3.3 4.7 6.8 10 15 22 33 47 68 100 150 220 330 470 680 1000

JA RR
8-10-77
LOW-COST SOLAR ARRAY PROJECTS

I-V CURVE CALCULATION FOR SERIES PARALLEL NETWORKS

- For elements in series add voltages along constant current lines

- For elements in parallel add currents along constant voltage lines
LOW-COST SOLAR ARRAY PROJECT

SIMULATION OF CUMULATIVE IV CURVE FOR A BRANCH CIRCUIT CONTAINING FAILED AND UNFAILED ELEMENTS

\[ I \text{ vs } V \]

\[ \text{failed cell} \]

\[ B + D + F \]

\[ (B + D + F) + (A + E) + C \]

CCG
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LOW-COST SOLAR ARRAY PROJECT
FAILURE PROBABILITIES:
THE STATISTICAL APPROACH

• OBJECTIVE

DETERMINE DISCRETE FAILURE STATES IN LARGE SYSTEMS WHICH ARE PROBABILISTIC IN NATURE

• APPROACH

USE BINOMIAL STATISTICS
YIELDS EXACT PROBABILITIES
ASSUMES INDEPENDENCE
LOW-COST SOLAR ARRAY PROJECT

FAILURE PROBABILITIES:
THE STATISTICAL APPROACH

- THE BINOMIAL EQUATION

\[ P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \]

WHERE:

\[ \binom{n}{k} \] IS THE BINOMIAL COEFFICIENT = \( \frac{n!}{k! (n-k)!} \)

- EXAMPLE

WE HAVE 6 COINS THAT CAN FALL INTO 3 BOXES WITH EQUAL
PROBABILITY \( p = \frac{1}{3} \). ONE BOX IS MARKED "HIT", THE OTHER
TWO "MISS"

<table>
<thead>
<tr>
<th>COINS =</th>
<th>BOXES =</th>
<th>MISS</th>
<th>HIT</th>
<th>MISS</th>
</tr>
</thead>
<tbody>
<tr>
<td>oo oo</td>
<td>oo oo</td>
<td>p = 1/3</td>
<td>1/3</td>
<td>1/3</td>
</tr>
</tbody>
</table>
LOW-COST SOLAR ARRAY PROJECT

FAILURE PROBABILITIES:
THE STATISTICAL APPROACH

EXAMPLE (CONT'D)

\[ P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \]

QUESTION: WHAT ARE THE PROBABILITIES ASSOCIATED WITH FINDING \( k \) COINS IN THE "HIT" BOX FOR \( k = 0, 1, 2, \ldots 6 \)?

SO

\( n = 6 \), NUMBER OF COINS

\( p = \frac{1}{3} \), PROBABILITY OF ANY COIN LANDING IN THE "HIT" BOX

\( k = 0 \) THRU 6, THE EXPECTED NUMBER OF COINS IN THE "HIT" BOX

\( P(X = k) = \) THE PROBABILITY OF FINDING EXACTLY \( k \) COINS IN THE "HIT" BOX
LOW-COST SOLAR ARRAY PROJECT
FAILURE PROBABILITIES:
THE STATISTICAL APPROACH

EXAMPLE (CONT'D)

\[ P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \]

<table>
<thead>
<tr>
<th>k</th>
<th>P(X = k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0878</td>
</tr>
<tr>
<td>1</td>
<td>0.2634</td>
</tr>
<tr>
<td>2</td>
<td>0.3292</td>
</tr>
<tr>
<td>3</td>
<td>0.2195</td>
</tr>
<tr>
<td>4</td>
<td>0.0824</td>
</tr>
<tr>
<td>5</td>
<td>0.0165</td>
</tr>
<tr>
<td>6</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

1.0000

QUESTION:
WHAT IS THE PROBABILITY OF FINDING AT LEAST ONE COIN IN THE "HIT" BOX?

ANSWER:
1 MINUS THE PROBABILITY OF FINDING NONE. \[ 1 - 0.0878 = 0.9122 \]
LOW-COST SOLAR ARRAY PROJECT

FAILURE PROBABILITIES:
THE STATISTICAL APPROACH

PV EXAMPLE:

A SUBSTRING = SIX CELLS IN SERIES

A SERIES BLOCK = FOUR SUBSTRINGS

A BRANCH CIRCUIT = 20 SERIES BLOCKS

CELLS FAIL AT THE RATE OF ONE PER 100
LOW-COST SOLAR ARRAY PROJECT
FAILURE PROBABILITIES:
THE STATISTICAL APPROACH

CELLS AND SUBSTRINGS

\[ P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \]

ASSUME:

- \( p = \text{CELL FAILURE RATE} = 0.01 \)
- \( n = 6, \text{CELLS IN A SUBSTRING} \)
- ONE FAILED CELL RESULTS IN A FAILED SUBSTRING
- MORE THAN ONE FAILED CELL HAS NO FURTHER EFFECT

THEREFORE:

SUBSTRING FAILURE RATE = 1 - PROBABILITY OF NO CELLS FAILED (i.e., \( k = 0 \)) WHERE THE PROBABILITY OF NO CELLS FAILED IS

\[ P(X = 0) = (1 - p)^n = (1 - 0.01)^6 = 0.9415 \]

SUBSTRING FAILURE RATE = 1 - 0.9415 = 0.0585

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FAILURE PROBABILITIES: THE STATISTICAL APPROACH

SERIES BLOCKS

\[ P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \]

A SERIES BLOCK IS A GROUP OF SUBSTRINGS WIRED IN PARALLEL

ASSUME:

- FOUR SUBSTRINGS PER SERIES BLOCK = n
- p IS NOW SUBSTRING FAILURE RATE = 0.0585
- k IS NOW THE NUMBER OF FAILED SUBSTRINGS

QUESTION:

WHAT ARE THE PROBABILITIES ASSOCIATED WITH FINDING ZERO TO FOUR SUBSTRINGS FAILED IN A SERIES BLOCK?
# Low-Cost Solar Array Project

## Failure Probabilities: The Statistical Approach

**Series Blocks (Cont'd)**

**Answer:**

<table>
<thead>
<tr>
<th>$k$ Failed Substrings</th>
<th>$P(X = k)$</th>
<th>Probability Of</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.7857</td>
<td>None Across Failed</td>
</tr>
<tr>
<td>1</td>
<td>0.1953</td>
<td>One Across Failed</td>
</tr>
<tr>
<td>2</td>
<td>0.0182</td>
<td>Two Across Failed</td>
</tr>
<tr>
<td>3</td>
<td>0.0007</td>
<td>Three Across Failed</td>
</tr>
<tr>
<td>4</td>
<td>0.0001</td>
<td>Four Across Failed</td>
</tr>
<tr>
<td></td>
<td>1.0000</td>
<td></td>
</tr>
</tbody>
</table>
LOW-COST SOLAR ARRAY PROJECT

FAILURE PROBABILITIES:

THE STATISTICAL APPROACH

BRANCH CIRCUIT

\[ P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \]

n IS NOW 20, THE NUMBER OF SERIES BLOCKS PER BRANCH CIRCUIT

QUESTION:

WHAT ARE THE PROBABILITIES ASSOCIATED WITH FINDING k SERIES BLOCKS WITH TWO ACROSS FAILED?

NOW

\[ p = 0.0182 \text{ (FROM PREVIOUS CALCULATIONS) AND} \]

\[ k = 0 \text{ TO } 20 \]

<table>
<thead>
<tr>
<th>k</th>
<th>PROBABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.6926</td>
</tr>
<tr>
<td>1</td>
<td>0.2568</td>
</tr>
<tr>
<td>2</td>
<td>0.0452</td>
</tr>
<tr>
<td>3</td>
<td>0.0050</td>
</tr>
<tr>
<td>4</td>
<td>0.0004</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
BRANCH CIRCUIT (CONTD)

• QUESTION:

WHAT IS THE PROBABILITY OF FINDING AT LEAST ONE SERIES BLOCK WITH TWO ACROSS SUBSTRINGS FAILED?

• ANSWER:

ONE MINUS THE PROBABILITY OF NONE WITH TWO ACROSS FAILED

\[ 1 - 0.6926 = 0.3074 \]

• THE ABOVE CAN BE FOUND FOR 0, 1, 3 AND 4 ACROSS FAILED IF DESIRED
CONCLUSION

BINOMIAL STATISTICS CAN BE USED TO DETERMINE THE DISCRETE FAILURE STATES OF LARGE SYSTEMS
LOW-COST SOLAR ARRAY PROJECT

MODULE DESIGN CONSTRAINTS

OBJECTIVE: ADDRESS CIRCUIT DESIGN CONSTRAINTS

ASSOCIATED WITH:

- MECHANICAL CONFIGURATION
  - GLASS COST vs MODULE SIZE
  - SUPPORT STRUCTURE COST vs MODULE SIZE
  - AESTHETICS

- SAFETY
  - MAXIMUM MODULE OPEN CIRCUIT VOLTAGE
  - MODULE VOLTAGE ISOLATION REQUIREMENT
LOW-COST SOLAR ARRAY PROJECT

GLASS COST vs MODULE AREA

- 50 lb/ft² LOADING
- 1% PROBABILITY OF FAILURE
- 15 min LOAD DURATION
- 0.01% IRON CONTENT

LENGTH/WIDTH RATIO

- a/b = 2
- a/b = 1
- a/b = 4

- 0.125" TEMPERED GLASS
- 0.125" ANNEALED GLASS
- 33" x 33" x 0.125" ANNEALED GLASS SUPERSTRATE YIELDS LEAST GLASS COST
LOW-COST SOLAR ARRAY PROJECT

PANEL STRUCTURE COST vs MODULE SIZE

(INTERMEDIATE SUPPORT POINTS)

4' x 8' PANELS

8' x 16' PANELS

2' x 4' MODULES

4' x 4' MODULES

4' x 8' MODULES

4' x 8' MODULES

2' x 4' MODULES

2' x 4' MODULES

4' x 4' MODULES

4' x 4' MODULES

4' x 8' MODULES

4' x 8' MODULES

PANEL COST (1975 $/m²)

0 10 20 30

0 50 25

20 15

8' x 16' PANELS

4' x 8' PANELS

LOADING (PSF)

50

2' x 4' 4' x 4'

4' x 8'

35

10 20 30

2

3

2

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LOW-COST SOLAR ARRAY PROJECT

PANEL STRUCTURE COST vs MODULE SIZE
(END SUPPORT POINTS)

<table>
<thead>
<tr>
<th>Panel Size</th>
<th>Modules</th>
</tr>
</thead>
<tbody>
<tr>
<td>4' x 8'</td>
<td>2' x 4'</td>
</tr>
<tr>
<td></td>
<td>4' x 4'</td>
</tr>
<tr>
<td></td>
<td>4' x 8'</td>
</tr>
<tr>
<td>8' x 16'</td>
<td>2' x 4'</td>
</tr>
<tr>
<td></td>
<td>4' x 4'</td>
</tr>
</tbody>
</table>

Panel Cost (1975 $/m²):
- 8' x 16' Panels:
  - 50
  - 35

4' x 8' Panels:
- 35

LOADING (PSF):
- 50
- 35

MODULE SIZE:
- 2' x 4'
- 4' x 4'
- 4' x 8'

0 10 20 30 FT²

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LOW-COST SOLAR ARRAY PROJECT

MODULE AESTHETIC CONSIDERATIONS

- SIZE: AS SMALL AS POSSIBLE
- TEXTURE: MATTE - LIKE
- PATTERN: RECTANGULAR, SUBDUED
- COLOR: EARTH - TONE
LOW-COST SOLAR ARRAY PROJECT

MAXIMUM MODULE OPEN CIRCUIT VOLTAGE

BACKGROUND: NATIONAL ELECTRICAL CODE IMPOSES SAFETY HANDLING REQUIREMENTS ON EQUIPMENT WITH MAXIMUM VOLTAGES \( \geq 30 \text{ VDC} \)

MAXIMUM VOLTAGE CONDITIONS FOR MODULES:

- CELL TEMPERATURE: \(-20^\circ\text{C}\)
- LOAD: OPEN CIRCUIT
- IRRADIANCE: \(100 \text{ mW/cm}^2\)

CONCLUSION: MODULE COST PENALTY FOR \(V_{OC} \geq 30 \text{ VDC}\)

WHEN \(S = 100\text{ mW/cm}^2, T_{CELL} = -20^\circ\text{C}\)
LOW-COST SOLAR ARRAY PROJECT

MODULE VOLTAGE ISOLATION REQUIREMENT

BACKGROUND: STANDARD TEST VOLTAGE FOR SAFETY

VERIFICATION IS TWICE MAXIMUM WORKING VOLTAGE + 1000 VOLTS

MAXIMUM WORKING VOLTAGE FOR MODULES:

- OPEN CIRCUIT SYSTEM VOLTAGE ABOVE GROUND
- CELL TEMPERATURE: 0°C
- IRRADIANCE: 100mW/cm²
LOW-COST SOLAR ARRAY PROJECT

MODULE DIELECTRIC TEST VOLTAGES

MAXIMUM POWER SYSTEM VOLTAGE AT NOCT

OPEN CIRCUIT SYSTEM VOLTAGE AT 0°C

HI-POT TEST VOLTAGE

CURRENT

NOCT 0°C

VOLTAGE

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CONCLUSIONS

• MODULE SIZE CONSIDERATIONS
  • GLASS COST/m² ABOUT EQUAL FOR ALL MODULE SIZES
  • MODULE SUPPORT COST/m² DECREASES AS MODULE SIZE INCREASES
  • RECTANGULAR SHAPE DESIREABLE

• ELECTRICAL SAFETY CONSIDERATIONS
  • MODULE COST PENALTY FOR $V_{OC}$ $(100 \text{mW/cm}^2, -20^\circ \text{C}) \geq 30$ VOLTS
  • SYSTEM DIELECTRIC TEST VOLTAGE EQUAL TO $2 \times \text{SYSTEM VOLTAGE AT 0}^\circ \text{C} + 1000$
LOW-COST SOLAR ARRAY PROJECT

MISMATCH LOSSES

- PROBLEM STATEMENT
  - REDUCE ELECTRICAL LOSSES DUE TO CELL MISMATCH WITHIN MODULES

- APPROACH
  - INTRODUCE CIRCUIT REDUNDANCY TO REDUCE MISMATCH LOSSES TO ACCEPTABLE LEVEL (< 5%)

- ANALYTICAL PROCEDURE
  - DETERMINE $I_{SC}$ DISTRIBUTION
  - USE MONTE CARLO TECHNIQUES TO SELECT $I_{SC}$ IN RANDOM WAY
  - COMBINE CELL IV CURVES TO COMPUTE LOSSES
LOW-COST SOLAR ARRAY PROJECT

MISMATCH LOSSES

- SIMULATED BY USING RANDOM DISTRIBUTION OF $I_{SC}$ AND FF
- $I_{SC}$

- MISMATCH EFFECTS MOST SEVERE WHERE COMBINING ALONG CONSTANT CURRENT LINES
- FF

- MISMATCH EFFECTS LESS SEVERE THAN THOSE DUE TO VARIATION IN $I_{SC}$
LOW-COST SOLAR ARRAY PROJECT

EFFECT OF MISMATCH ON MODULE POWER

A  I-V CURVE WITH NO MISMATCH IN $I_{SC}$

B  I-V CURVE WITH MISMATCH IN $I_{SC}$

MAX POWER POINT
LOW-COST SOLAR ARRAY PROJECT

MODULE MISMATCH vs CELL $I_{SC}$

DISTRIBUTION AND MODULE SERIES/PARALLELING

![Graph showing the fractional decrease in power vs number of parallel strings and $I_{SC}$ distribution.]

- **Fractional Decrease in Power (%)**
  - Y-axis ranges from 0 to 10%
- **Number of Parallel Strings**
  - X-axis ranges from 1 to 8, labeled as 1, 2, 3, 4, 5, 6, 7, 8
- **Series Blocks per Module**
  - X-axis ranges from 1 to 8
- **$I_{SC}$ Distribution**
  - Y-axis labeled as Fraction of Total Cells
  - Data points for 5%, 9%, and AVG. (average)
LOW-COST SOLAR ARRAY PROJECT

MODULE MISMATCH vs CELL \( I_{SC} \) DISTRIBUTION AND MODULE SERIES/PARALLELING

- **Fractional Decrease in Power (%):**
  - Series Blocks per Module:
    - 1
    - 2
    - 4
    - 8
    - 12
    - 16

- **Number of Parallel Strings:**

- **Fraction of Total Cells:**
  - \( I_{SC} \) Distribution:
    - Avg. +15%
    - S = 9%
    - -21%

2-5
LOW-COST SOLAR ARRAY PROJECT

MODULE MISMATCH vs CELL $I_{SC}$ DISTRIBUTION
AND MODULE SERIES/PARALLELING

[Graph showing the relationship between number of parallel strings and fractional decrease in power.]

[Bar chart showing the fraction of total cells with different $I_{SC}$ distributions, indicating $S = 9\%$.]

SERIES BLOCKS PER MODULE

FRACTIONAL DECREASE IN POWER, %
LOW-COST SOLAR ARRAY PROJECT

MISMATCH LOSSES CONCLUSIONS

- Key factors determining amount of mismatch
  - $I_{SC}$ distribution shape and half-width
  - Ratio of $I_{MAX \, POWER}$ to $I_{SC}$
  - Cell shunt resistance

- Mismatch losses at max power not significant for 10 percent $I_{SC}$ half widths

- Due to short circuit current loss, mismatch losses at other than max power may be significant

- Hot-spot heating problems may develop from mismatch - but
LOW-COST SOLAR ARRAY PROJECT

MODULE MANUFACTURING YIELD

PROBLEM STATEMENT

• REDUCE NUMBER OF MODULE REJECTS DUE TO FAILURES DURING MODULE ASSEMBLY, SHIPPING AND INSTALLATION

APPROACH

• INTRODUCE CIRCUIT REDUNDANCY TO REDUCE SINGLE-FAILURE POWER LOSS BELOW ACCEPTABLE LEVEL (10%)
  • MULTIPLE CELL CONTACTS
  • SERIES/PARALLELING
  • BYPASS DIODES
LOW-COST SOLAR ARRAY PROJECT

MODULE MANUFACTURING YIELD ANALYTICAL PROCEDURE

- DETERMINE CELL FAILURE DENSITY INCLUDING EFFECTS OF MULTIPLE CONTACTS

- CALCULATE FRACTION OF MODULES HAVING A GIVEN NUMBER OF FAILED CELLS

- DETERMINE MODULE POWER LOSS FOR EACH NUMBER OF FAILED CELLS INCLUDING EFFECT OF SERIES/PARALLEL/DIODES.

- CALCULATE TOTAL FRACTION OF MODULES WITH UNACCEPTABLE POWER LOSS (GREATER THAN 10%)
LOW-COST SOLAR ARRAY PROJECT
MULTIPLE CELL CONTACTS

- PROBLEM STATEMENT
  - REDUCE CELL AREA LOSS DUE TO CRACKING

- APPROACH
  - INTRODUCE REDUNDANT CONTACTS TO REDUCE CELL AREA LOSS TO LESS THAN 10%

- ANALYTICAL PROCEDURE
  - DETERMINE AREA LOSS FOR LARGE NUMBER OF RANDOMLY SELECTED CRACKS (SITE AND DIRECTIONS) USING MONTE CARLO TECHNIQUE.
  - CALCULATE FRACTION OF CRACKS CAUSING UNACCEPTABLE AREA LOSS (e.g. GREATER THAN 10%)
MULTIPLE CELL CONTACTS-
ANALYTICAL PROCEDURE
Low-Cost Solar Array Project

Fraction of Cracked Cells Leading to Failed Cells for Various Multiple Cell Contacts

<table>
<thead>
<tr>
<th>Percent Loss of Cell Area</th>
<th>$0^\circ$</th>
<th>$60^\circ$</th>
<th>$90^\circ$</th>
<th>$-180^\circ$</th>
<th>$+$</th>
<th>$+$ $-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>.36</td>
<td>.42</td>
<td>.50</td>
<td>.64</td>
<td>.36</td>
<td>.91</td>
</tr>
<tr>
<td>5-10</td>
<td>.09</td>
<td>.15</td>
<td>.18</td>
<td>.18</td>
<td>.14</td>
<td>.09</td>
</tr>
<tr>
<td>10-20</td>
<td>.06</td>
<td>.12</td>
<td>.12</td>
<td>.12</td>
<td>.12</td>
<td>0</td>
</tr>
<tr>
<td>20-40</td>
<td>.03</td>
<td>.06</td>
<td>.06</td>
<td>.06</td>
<td>.11</td>
<td>0</td>
</tr>
<tr>
<td>40-70</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>.45</td>
<td>.24</td>
<td>.14</td>
<td>0</td>
<td>.23</td>
<td>0</td>
</tr>
<tr>
<td>SUM of ≥10</td>
<td>.54</td>
<td>.42</td>
<td>.32</td>
<td>.18</td>
<td>.50</td>
<td>0</td>
</tr>
</tbody>
</table>
LOW-COST SOLAR ARRAY PROJECT

MULTIPLE CELL CONTACT CONCLUSIONS

- Single contacts lead to a 50% failure rate among cracked cells
- Use of double tabs reduces failure rate by 20%-60%, depending on orientation
- Use of triple tabs leads to negligible failure rate
LOW-COST SOLAR ARRAY PROJECT

MODULE MANUFACTURING YIELD

ANALYTICAL PROCEDURE

- Determine cell failure density including effects of multiple contacts
- Calculate fraction of modules having a given number of failed cells
- Determine module power loss for each number of failed cells including effect of series/parallel/diodes.
- Calculate total fraction of modules with unacceptable power loss (greater than 10%)
LOW-COST SOLAR ARRAY PROJECT

FRACTION OF MODULES WITH X FAILED CELLS

ANALYTICAL PROCEDURE:

\[ f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \approx \frac{e^{-\mu} \mu^x}{x!} \]

WHERE:

- \( f(x) \) = FRACTION OF MODULES WITH X FAILED CELLS
- \( \mu = nP \)
- \( n = NUMBER \ OF \ CELLS \ PER \ MODULE \)
- \( p = FRACTION \ OF \ FAILED \ CELLS \ IN \ CELL \ POPULATION \)

EXAMPLE (\( p = .001 \)):

<table>
<thead>
<tr>
<th>( n )</th>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>( f(x) )</td>
<td>.905</td>
<td>.0905</td>
<td>.00452</td>
<td>.00015</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>.819</td>
<td>.164</td>
<td>.0164</td>
<td>.00109</td>
</tr>
<tr>
<td>400</td>
<td></td>
<td>.670</td>
<td>.268</td>
<td>.0536</td>
<td>.00715</td>
</tr>
</tbody>
</table>

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LOW-COST SOLAR ARRAY PROJECT
MODULE MANUFACTURING YIELD
ANALYTICAL PROCEDURE

- DETERMINE CELL FAILURE DENSITY
  INCLUDING EFFECTS OF MULTIPLE CONTACTS

- CALCULATE FRACTION OF MODULES
  HAVING A GIVEN NUMBER OF FAILED CELLS

- DETERMINE MODULE POWER LOSS FOR
  EACH NUMBER OF FAILED CELLS INCLUDING
  EFFECT OF SERIES/PARALLEL/DIODES.

- CALCULATE TOTAL FRACTION OF
  MODULES WITH UNACCEPTABLE POWER
  LOSS (GREATER THAN 10%)
LOW-COST SOLAR ARRAY PROJECT

MODULE POWER LOSS CALCULATION

ANALYTICAL PROCEDURE

• GENERATE I-V CURVE FOR FAILED AND UNFAILED MODULE (COMPUTER).

• CALCULATE POWER LOSS ASSUMING EITHER MAXIMUM POWER OR CONSTANT VOLTAGE OPERATION AS APPROPRIATE.
LOW-COST SOLAR ARRAY PROJECT

EFFECT OF SERIES PARALLELING ON A MODULE
WITH A FAILED CELL

SAMPLE CASES ANALYZED

FAIRED CELLS

12 CELLS

SERIES BLOCKS PER MODULE

NO FAILED CELLS

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LOW-COST SOLAR ARRAY PROJECT

I-V CURVE FOR CASE OF ONE BAD CELL USING A BY-PASS DIODE

WITH A BY-PASS DIODE

WITHOUT A BY-PASS DIODE

MAX POWER POINT WITH A BY-PASS DIODE

MAX POWER POINT WITHOUT A BY-PASS DIODE

SIX SERIES BLOCKS TWO PARALLEL STRINGS
LOW-COST SOLAR ARRAY PROJECT
TWO ALTERNATE SERIES/PARALLEL/DIODE COMBINATIONS
LOW-COST SOLAR ARRAY PROJECT
MODULE POWER LOSS VERSUS MODULE SERIES/PARALLELING
(1 TO 3 FAILED CELLS PER MODULE)

FRACTION MODULE POWER LOSS

SERIES BLOCKS PER MODULE

1, 2, 4 PARALLEL STRINGS BASED ON 1 CELL FAILURE ONLY
LOW-COST SOLAR ARRAY PROJECT

MODULE MANUFACTURING YIELD
ANALYTICAL PROCEDURE

- DETERMINE CELL FAILURE DENSITY
  INCLUDING EFFECTS OF MULTIPLE CONTACTS

- CALCULATE FRACTION OF MODULES
  HAVING A GIVEN NUMBER OF FAILED CELLS

- DETERMINE MODULE POWER LOSS FOR
  EACH NUMBER OF FAILED CELLS INCLUDING
  EFFECT OF SERIES/PARALLEL/DIODES.

- CALCULATE TOTAL FRACTION OF
  MODULES WITH UNACCEPTABLE POWER
  LOSS (GREATER THAN 10%)
LOW-COST SOLAR ARRAY PROJECT

MODULE MANUFACTURING YIELD EXAMPLE

EXAMPLE CONFIGURATION:
96 CELLS PER MODULE
6 PARALLEL BY 4 SERIES BLOCKS
CELL FAILURE DENSITY = 0.001

<table>
<thead>
<tr>
<th>FAILED CELLS PER MODULE</th>
<th>FRACTION MODULES</th>
<th>MODULE FRACTION POWER LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.9050</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.0905</td>
<td>0.07</td>
</tr>
<tr>
<td>2</td>
<td>0.0045</td>
<td>0.11</td>
</tr>
<tr>
<td>3</td>
<td>0.0001</td>
<td>0.15</td>
</tr>
</tbody>
</table>

FRACTION MODULES WITH <10% POWER LOSS = 0.905 + 0.0905 = [0.995]

= 1 - (0.0045 - 0.0001)
LOW-COST SOLAR ARRAY PROJECT
MANUFACTURING YIELD DUE TO CELL BREAKAGE
VERSUS MODULE SERIES/PARALLELING
(CELL BREAKAGE = 1 PER 1000 REJECTION CRITERIA: P < 0.9 P AVG)

100 CELL MODULE

200 CELL MODULE

400 CELL MODULE

PARALLEL STRINGS

PARALLEL STRINGS

PARALLEL STRINGS

4 OR LESS PARALLEL STRINGS

4 OR LESS PARALLEL STRINGS

4 OR LESS PARALLEL STRINGS

SERIES BLOCKS PER MODULE

SERIES BLOCKS PER MODULE

SERIES BLOCKS PER MODULE

YIELD

YIELD

YIELD

99.99
99.99
99.99

99.9
99.9
99.9

99
99
99

99.99
99.99
99.99

99
99
99

95
95
95

90
90
90

80
80
80

70
70
70

60
60
60

50
50
50

0
15
20
0
15
20
0
15
20

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MANUFACTURING YIELD DUE TO CELL BREAKAGE vs MODULE SERIES/PARALLELING

(CELL BREAKAGE = 1 PER 100
REJECTION CRITERIA: P < 0.9 PAVG)

100 CELL MODULE

<table>
<thead>
<tr>
<th>SERIES BLOCKS PER MODULE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>99.99</td>
</tr>
<tr>
<td>16</td>
</tr>
</tbody>
</table>

200 CELL MODULE

<table>
<thead>
<tr>
<th>SERIES BLOCKS PER MODULE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>99.99</td>
</tr>
<tr>
<td>16</td>
</tr>
</tbody>
</table>

400 CELL MODULE

<table>
<thead>
<tr>
<th>SERIES BLOCKS PER MODULE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>99.99</td>
</tr>
<tr>
<td>16</td>
</tr>
</tbody>
</table>

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LOW-COST SOLAR ARRAY PROJECT
MANUFACTURING YIELD DUE TO CELL BREAKAGE vs MODULE SERIES/PARALLELING
(CELL BREAKAGE = 1 PER 10000 REJECTION CRITERIA: $P < 0.9 P_{AVG}$)
LOW-COST SOLAR ARRAY PROJECT

MANUFACTURING YIELD vs CELL FAILURE FRACTION
EIGHT PARALLEL STRINGS/TWO SERIES BLOCKS

CELLS / MODULE
- - - - - - 100
- - - - - - 200
- - - - - - 400

CELL FAILURE FRACTION (LOG10)
LOW-COST SOLAR ARRAY PROJECT

MANUFACTURING YIELD vs CELL FAILURE FRACTION

FOUR PARALLEL STRINGS

CELLS / MODULE
- - - 100
- - 200
- - - 400

CELL FAILURE FRACTION (\log_{10})

MANUFACTURING YIELD

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MANUFACTURING YIELD vs CELL FAILURE FRACTION

EIGHT PARALLEL STRINGS/ONE SERIES BLOCK

![Graph showing the relationship between manufacturing yield and cell failure fraction. The x-axis represents the cell failure fraction (log10) and the y-axis represents the manufacturing yield. The graph includes lines for different numbers of cells per module: 100, 200, and 400.](image-url)
LOW-COST SOLAR ARRAY PROJECT

MANUFACTURING YIELD vs CELL FAILURE FRACTION

EIGHT PARALLEL STRINGS/THREE SERIES BLOCKS

MANUFACTURING YIELD vs CELL FAILURE FRACTION

CELLS / MODULE
100
200
400

CELL FAILURE FRACTION (LOG10)
LOW-COST SOLAR ARRAY PROJECT

MANUFACTURING YIELD vs CELL FAILURE FRACTION
EIGHT PARALLEL STRINGS / FIVE SERIES BLOCKS

MANUFACTURING YIELD

CELL FAILURE FRACTION (log_{10})

CELLS / MODULE
- - - - 100
- - - - 200
- - - - 400

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MANUFACTURING YIELD vs CELL FAILURE FRACTION
EIGHT PARALLEL STRINGS/SIX SERIES BLOCKS

CELLS / MODULE
- - - - 100
- - - 200
- - - - 400

MANUFACTURING YIELD

CELL FAILURE FRACTION (LOG10)

50
60
70
80
90
99
99.9
99.99

-1
-2
-3
-4
-5
-6
LOW-COST SOLAR ARRAY PROJECT

MANUFACTURING YIELD vs CELL FAILURE FRACTION

SIXTEEN PARALLEL STRINGS/ONE SERIES BLOCK

MANUFACTURING YIELD

CELL FAILURE FRACTION (LOG10)

CELLS / MODULE
- 100
- 200
- 400

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LOW-COST SOLAR ARRAY PROJECT

MANUFACTURING YIELD vs CELL FAILURE FRACTION

SIXTEEN PARALLEL STRINGS/TWO SERIES BLOCKS

CELLS / MODULE
- 100
- 200
- 400

CELL FAILURE FRACTION (LOG10)
LOW-COST SOLAR ARRAY PROJECT

MANUFACTURING YIELD vs CELL FAILURE FRACTION

SIXTEEN PARALLEL STRINGS/THREE SERIES BLOCKS

MANUFACTURING YIELD

CELL FAILURE FRACTION (LOG10)

CELLS / MODULE

100

200

400

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LOW-COST SOLAR ARRAY PROJECT
MANUFACTURING YIELD CONCLUSIONS

- For modules of 100 cells or larger with four parallel strings, yield is 90% or less and is independent of number of series blocks.

- For modules with 6 parallel strings, yield can be increased to 99% by adding up to 6 series blocks.

- For modules with 8 or more parallel strings, yield can be increased to 99% by adding up to 3 series blocks.
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VISUALIZATION OF
"HOT SPOT" CELL HEATING

FAILED CELL

POWER DISSIPATION IN B

A

A + B

B

V

I
LOW-COST SOLAR ARRAY PROJECT

HOT-SPOT HEATING OF CRACKED CELLS

MAXIMUM HEATING AT DIODE TURN-ON

\[ \frac{P}{P_{\text{MAX}}} \approx 10 \]

\[ \text{CRACKED CELL} \]

CRACKED CELL IS REVERSE BIASED AT VOLTAGES BELOW THIS LEVEL

\[ A + B \]

\[ A \]

\[ B \]
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TYPICAL HOT-SPOT HEATING LEVEL
FOR FLAT-PLATE MODULE

HOT-SPOT POWER DISSIPATION ($P/P_{\text{MAX}}$)

MEASURED DATA
FOR GLASS MODULE
WITH 4 IN. CELLS

$T_{\text{CELL}} - T_{\text{AIR}}$, °C

TOTAL POWER INTO HOT-SPOT REGION, mW/cm²

100 mW/cm² IRRADIANCE + ELECTRICAL POWER

IRRADIANCE ONLY ($\alpha = 0.85$)

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EFFECT OF SERIES/PARALLELING ON HOT SPOT CELL HEATING

--- (SHUNT RESISTANCE = 100Ω)

----- (SHUNT RESISTANCE = 10Ω)
LOW-COST SOLAR ARRAY PROJECT

HOT-SPOT HEATING CONCLUSIONS

• HOT-SPOT HEATING LEVEL DEPENDENT ON:
  • SERIES BLOCKS PER DIODE (FEWER IS BETTER)
  • PARALLEL STRINGS (MORE IS BETTER)
  • CELL SHUNT RESISTANCE (APPLICATION SPECIFIC)
  • SYSTEM OPERATING POINT (SHORT CIRCUIT IS WORST)

• CRACKED CELLS OFTEN PRESENT WORST-CASE HEATING LEVEL:
  • 1/2 CELL ≈ 2 PARALLEL, 2/3 ≈ 3 PARALLEL, ETC.

• SERIES BLOCKS = SERIES CELLS

• GENERAL DESIGN RULE:
  • NO MORE THAN 15 SERIES CELLS PER DIODE IF CRACKED CELL CAN LOSE ≥ 10% OF AREA

  • NO MORE THAN 15 SERIES BLOCKS PER DIODE

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LOW-COST SOLAR ARRAY PROJECT

ARRAY ARCING CONSIDERATIONS

PROBLEM: STEADY STATE ARCS CAN FORM WHEN OPEN CIRCUITS DEVELOP IN HIGH VOLTAGE BRANCH CIRCUITS.

SOLUTION: MINIMIZE CHANCE OF OPEN CIRCUITS THROUGH USE OF CIRCUIT REDUNDANCY.
LOW-COST SOLAR ARRAY PROJECT
CRACKED AND FAILED CELLS
DUE TO FIELD EXPOSURE

<table>
<thead>
<tr>
<th>SITE</th>
<th>TOTAL NUMBER OF CELLS IN FIELD</th>
<th>FRACTION(^\circ) CRACKED PER YEAR</th>
<th>FRACTION FAILED PER YEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAD, NEBRASKA</td>
<td>90,168</td>
<td>0.010</td>
<td>0.00021</td>
</tr>
<tr>
<td>MT. LAGUNA, CALIF.</td>
<td>96,236</td>
<td>0.025</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

*30 TO 50% DUE TO HAIL IMPACT
LOW-COST SOLAR ARRAY PROJECT
CRACKED CELLS IN MODULES
AT FINAL MODULE INSPECTION

<table>
<thead>
<tr>
<th>MODULE TYPE</th>
<th>TOTAL CELLS IN BUY</th>
<th>FRACTION CRACKED</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLOCK II</td>
<td>252,070</td>
<td>0.0004 - 0.02</td>
</tr>
<tr>
<td>BLOCK III</td>
<td>158,048</td>
<td>0.0009 - 0.02</td>
</tr>
</tbody>
</table>
LOW-COST SOLAR ARRAY PROJECT
ARRAY FAULT TOLERANCE

• PROBLEM STATEMENT
  • REDUCE ARRAY POWER DEGRADATION DUE
    TO CELL AND MODULE FAILURES

• APPROACH
  • INCREASE ARRAY FAULT TOLERANCE
    BY PROVIDING REDUNDANT CURRENT PATHS
LOW-COST SOLAR ARRAY PROJECT

TECHNIQUES FOR ENHANCEMENT
OF FAULT TOLERANCE

- MULTIPLE CELL CONTACTS
- SERIES/PARALLELING
- USE OF BYPASS DIODES
LOW-COST SOLAR ARRAY PROJECT
PROCEDURE FOR DETERMINING ARRAY POWER DEGRADATION

- Compute substring failure density ($F_{SS}$) for given series/parallel configuration, cell failure density ($F_C$), and no. of cells per substring ($N$) using:
  \[ F_{SS} = 1 - (1 - F_C)^N \]

- Determine fraction of branch circuits with given levels of failed substrings

- Use computer model to calculate branch circuit power loss by combining I-V curves of:
  - Failed series blocks
  - Unfailed series blocks

- Combine power degradation of branch circuits to obtain system degradation

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SUBSTRING FAILURE DENSITY AS A FUNCTION OF CELL FAILURE DENSITY

CELL FAILURE DENSITY = 0.0001

SUBSTRING FAILURE DENSITY

CELLS PER SUBSTRING

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PROCEDURE FOR DETERMINING
ARRAY POWER DEGRADATION

- Compute substring failure density ($F_{SS}$) for given series/parallel configuration, cell failure density ($F_C$), and no. of cells per substring ($N$) using:
  \[ F_{SS} = 1 - (1 - F_C)^N \]

- Determine fraction of branch circuits with given levels of failed substrings

- Use computer model to calculate branch circuit power loss by combining I-V curves of:
  - Failed series blocks
  - Unfailed series blocks

- Combine power degradation of branch circuits to obtain system degradation

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BRANCH CIRCUIT FAILURE DISTRIBUTION

CATEGORIZE BRANCH CIRCUITS BY NUMBER OF FAILED SUBSTRINGS IN WORST CASE SERIES BLOCK

POWER CONDITIONER

FAILED CELLS

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BRANCH CIRCUIT FAILURE CATEGORIES

MAXIMUM NUMBER OF FAILED SUBSTRINGS PER SERIES BLOCK

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EXAMPLE BRANCH CIRCUIT
FAILURE DISTRIBUTION
(4 PARALLEL BY 50 SERIES BLOCKS)

FRACTION BRANCH CIRCUITS WITH MAXIMUM OF N FAILED SUBSTRINGS

SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

PROCEDURE FOR DETERMINING ARRAY POWER DEGRADATION

- Compute substring failure density ($F_{SS}$) for given series/parallel configuration, cell failure density ($F_C$), and no. of cells per substring ($N$) using:
  $$F_{SS} = 1 - (1 - F_C)^N$$

- Determine fraction of branch circuits with given levels of failed substrings

- Use computer model to calculate branch circuit power loss by combining 1-V curves of:
  - Failed series blocks
  - Unfailed series blocks

- Combine power degradation of branch circuits to obtain system degradation
LOW-COST SOLAR ARRAY PROJECT
SYSTEM SERIES BLOCK FAILURE
FRACTION vs SUBSTRING FAILURE
DENSITY/FOUR PARALLEL STRINGS

NUMBER OF FAILED STRINGS

FRACTION OF SERIES BLOCKS WITH X FAILED SUBSTRINGS ACROSS

SUBSTRING FAILURE DENSITY

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SYSTEM SERIES BLOCK FAILURE
FRACTION vs SUBSTRING FAILURE
DENSITY / EIGHT PARALLEL STRINGS

NUMBER OF FAILED SUBSTRINGS

FRACTION OF SERIES BLOCKS WITH X FAILED SUBSTRINGS ACROSS

SUBSTRING FAILURE DENSITY

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SYSTEM SERIES BLOCK FAILURE FRACTION vs SUBSTRING FAILURE DENSITY / SIXTEEN PARALLEL STRINGS

FRACTION OF SERIES BLOCKS WITH X FAILED SUBSTRINGS ACROSS

SUBSTRING FAILURE DENSITY

NUMBER OF FAILED STRINGS

SEE ENLARGEMENT

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BRANCH CIRCUIT POWER LOSS vs FRACTION OF FAILED SERIES BLOCKS,
FOUR PARALLEL STRINGS, WITHOUT DIODES

NOTE: POWER LOSS COMPUTED AS IF ONLY X SUBSTRINGS ARE FAILED

FILL FACTOR = 0.7
LOW-COST SOLAR ARRAY PROJECT

BRANCH CIRCUIT POWER LOSS vs FRACTION OF FAILED SERIES BLOCKS, EIGHT PARALLEL STRINGS, WITHOUT DIODES

NOTE: POWER LOSS COMPUTED AS IF ONLY X SUBSTRINGS ARE FAILED

FILL FACTOR = 0.7

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PROCEDURE FOR DETERMINING ARRAY POWER DEGRADATION

• COMPUTE SUBSTRING FAILURE DENSITY ($F_{SS}$) FOR GIVEN SERIES/PARALLEL CONFIGURATION, CELL FAILURE DENSITY ($F_c$), AND NO. OF CELLS PER SUBSTRING ($N$) USING:
  \[ F_{SS} = 1 - (1 - F_c)^N \]

• DETERMINE FRACTION OF BRANCH CIRCUITS WITH GIVEN LEVELS OF FAILED SUBSTRINGS

• USE COMPUTER MODEL TO CALCULATE BRANCH CIRCUIT POWER LOSS BY COMBINING I-V CURVES OF:
  • FAILED SERIES BLOCKS
  • UNFAILED SERIES BLOCKS

• COMBINE POWER DEGRADATION OF BRANCH CIRCUITS TO OBTAIN SYSTEM DEGRADATION
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BRANCH CIRCUIT FAILURE CATEGORIES

MAXIMUM NUMBER OF FAILED SUBSTRINGS PER SERIES BLOCK

EXAMPLE BRANCH CIRCUIT POWER LOSS FRACTION

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TECHNIQUE FOR DETERMINING ARRAY POWER LOSS

CONSIDER CASE OF $i = 2$, EACH OF THE $F_2$ BRANCH CIRCUITS INVOLVED WILL HAVE A GIVEN NUMBER $N_{20}$ SERIES BLOCKS WITH 0 FAILED SUBSTRINGS, $N_{21}$ SERIES BLOCKS WITH 1 FAILED SUBSTRING, AND $N_{22}$ SERIES BLOCKS WITH 2 FAILED SUBSTRINGS. $\sum N_{ij} = N$

$F_1$ BRANCH CIRCUITS WILL HAVE A MAXIMUM OF $i$ FAILED SUBSTRINGS WHERE $i = 0, 1, 2, 3, 4$ WHERE $\sum F_i = \text{TOTAL NUMBER OF BRANCH CIRCUITS.}$ EACH BRANCH CIRCUIT HAS $N$ SERIES BLOCKS.

$F_0$, $F_1$, $F_3$ BRANCH CIRCUITS

$F_0$, $F_1$, AND $F_3$ BRANCH CIRCUITS

$F_2$, $F_1$, AND $F_3$ BRANCH CIRCUITS

IV CURVE OF THE TYPICAL $F_2$ BRANCH CIRCUIT

IV CURVE OF THE TYPICAL $F_3$ BRANCH CIRCUIT

THE IV CURVES OF ALL $F_2$ BRANCH CIRCUITS COMBINE TO GIVE

THIS IV CURVE COMBINES WITH THOSE FROM THE $F_0$, $F_1$, AND $F_3$ BRANCH CIRCUITS

$F_0$, $F_1$, AND $F_3$ BRANCH CIRCUITS

ARRAY IV CURVE

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EXAMPLE ARRAY POWER LOSS CALCULATION

CONFIGURATION:
CIRCUIT = 4 PARALLEL BY 50 SERIES BLOCKS
SUBSTRING FAILURE DENSITY = 0.1

<table>
<thead>
<tr>
<th>MAXIMUM NUMBER SUBSTRINGS FAILED</th>
<th>FRACTION OF BRANCH CIRCUITS ((F_i))</th>
<th>POWER LOSS FRACTION ((P_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.069</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>0.762</td>
<td>0.43</td>
</tr>
<tr>
<td>3</td>
<td>0.164</td>
<td>0.71</td>
</tr>
<tr>
<td>4</td>
<td>0.005</td>
<td>1.00</td>
</tr>
</tbody>
</table>

ARRAY POWER LOSS FRACTION \(= \sum_{i=0}^{4} F_i \times P_i = 0.46\)
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ARRAY POWER LOSS

4 PARALLEL STRINGS
FF = 0.70
NO DIODES

FSS, SUBSTRING FAILURE DENSITY

SERIES BLOCKS PER BRANCH CIRCUIT

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OVERALL MODULE/ARRAY DESIGN OPTIMIZATION

OBJECTIVE: MINIMIZE ARRAY LIFE-CYCLE ENERGY COST

METHODOLOGY:

\[ \text{LIFE-CYCLE BENEFIT} = \text{LIFE-CYCLE COST} \]

\[ (\$/\text{kW-h}) \times \left( \frac{\text{LIFE-CYCLE}}{\text{ENERGY}} \right) = \text{LIFE-CYCLE COST} \]

THEREFORE:

OPTIMUM = MINIMUM (\$/\text{kW-h})

MODULE

= MINIMUM \left( \frac{\text{LIFE-CYCLE COST}}{\text{LIFE-CYCLE ENERGY}} \right)
LOW-COST SOLAR ARRAY PROJECT

OPTIMIZATION ALGORITHM

\[ \text{OPTIMUM} = \text{MINIMUM} \left( \frac{\text{LIFE-CYCLE COST}}{\text{LIFE-CYCLE ENERGY}} \right) \]

\[ = \text{MINIMUM} \left( \frac{C_0 + \sum_{n=1}^{L} C_n (1 + k)^{-n}}{E_0 \sum_{n=1}^{L} \epsilon_n (1 + k)^{-n}} \right) \]

**THEREFORE:**

\[ \text{OPTIMUM} = \text{MINIMUM} \left( \frac{(\text{BALANCE OF PLANT COST, $/kW}) + (\text{INITIAL ARRAY COST/m}^2 + \text{L-C O & M COST/m}^2)}{(\text{PLANT EFFICIENCY})/100 \text{mW/cm}^2, \text{NOCT}} \right) \frac{(\text{ANNUAL INSOLATION}) \times (\text{L-C ENERGY FRACTION})}{(\text{kW-h/m}^2/\text{yr}) \times (\text{FRACTION}^* \text{)})} \]

\[ \text{L-C ENERGY FRACTION} = \sum_{n=1}^{L} \left( \frac{\text{POWER IN YEAR n}}{\text{INITIAL POWER}} \right) (1 + k)^{-n}, \ k = \text{DISCOUNT RATE} \]
### LOW-COST SOLAR ARRAY PROJECT

#### NOMINAL ARRAY COSTS

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>UNITS (1975 $)</th>
<th>MODULE SIZE (ft x ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2 x 4</td>
</tr>
<tr>
<td><strong>INITIAL:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Module Direct Cost</td>
<td>$/m²</td>
<td>60</td>
</tr>
<tr>
<td>Module Yield Cost*</td>
<td>$/m²</td>
<td>0-5</td>
</tr>
<tr>
<td>• Module Subtotal</td>
<td>$/m²</td>
<td>60-65</td>
</tr>
<tr>
<td>Panel Frame</td>
<td>$/m²</td>
<td>24</td>
</tr>
<tr>
<td>Panel Wiring</td>
<td>$/m²</td>
<td>2-4</td>
</tr>
<tr>
<td>• Panel Subtotal</td>
<td>$/m²</td>
<td>26-28</td>
</tr>
<tr>
<td>Panel Installation</td>
<td>$/m²</td>
<td>1</td>
</tr>
<tr>
<td>Installed Array Struct</td>
<td>$/m²</td>
<td>22</td>
</tr>
<tr>
<td>• Array Total</td>
<td>$/m²</td>
<td>109-116</td>
</tr>
<tr>
<td><strong>PER REPLACEMENT ACTION:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fault Identification</td>
<td>$/Panel</td>
<td>4</td>
</tr>
<tr>
<td>Panel Substitution Labor</td>
<td>$/Panel</td>
<td>21</td>
</tr>
<tr>
<td>Module Replacement Labor</td>
<td>$/Mod</td>
<td>12</td>
</tr>
<tr>
<td>Replacement Module Parts</td>
<td>$/m²</td>
<td>61-66</td>
</tr>
</tbody>
</table>

*1 CELL FAILURE PER 1000 DURING ASSEMBLY/SHIPPING/INSTALLATION

RWW
4/1/80
### LOW-COST SOLAR ARRAY PROJECT

#### NOMINAL PERFORMANCE PARAMETERS

<table>
<thead>
<tr>
<th>MODULE SIZE (ft x ft)</th>
<th>2 x 4</th>
<th>4 x 4</th>
<th>4 x 8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INITIAL ARRAY EFFICIENCY</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENCAP. CELL EFFICIENCY</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>NOCT EFFICIENCY</td>
<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>PACKING EFFICIENCY</td>
<td>0.89</td>
<td>0.91</td>
<td>0.93</td>
</tr>
<tr>
<td>ARRAY EFFICIENCY SUBTOTAL</td>
<td>0.123</td>
<td>0.126</td>
<td>0.128</td>
</tr>
</tbody>
</table>

| **BALANCE-OF-PLANT EFFICIENCY** |       |       |       |
| ELECTRICAL EFFICIENCY       | 0.92  |       |       |
| MODULE SOILING EFFICIENCY   | 0.92  |       |       |
| BALANCE-OF-PLANT SUBTOTAL   | 0.85  |       |       |

| **BALANCE-OF-PLANT COSTS (1975$)** | 150 $/kW |
| **DISCOUNT RATE (OVER INFLATION)** | 10% |
| **ANNUAL INSOLATION** | 1825 kW-h/m²/yr |
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER DEGRADATION vs TIME

2400 CELLS

8 PARALLEL STRINGS
FF = 0.70
CFR = 0.0001
NO DIODES

15 SERIES BLOCKS PER BRANCH CIRCUIT

TIME (YEARS)
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS DEGRADATION vs TIME

1200 CELLS
8 PARALLEL STRINGS  CFR = .0001
FF = .70  NO DIODES

15 SERIES BLOCKS PER BRANCH CIRCUIT

FRACTION INITIAL POWER LOSS

TIME (YEARS)

CCG
3-31-80
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS DEGRADATION vs TIME

600 CELLS

8 PARALLEL STRINGS CFR = .0001

FF = .70 NO DIODES

1 SERIES BLOCKS PER BRANCH CIRCUIT

1.0

.5

0

FRACTION INITIAL POWER LOSS

0 5 10 15 20

TIME (YEARS)

600° 240° 120° 60° 30° 15°

CCG
3-31-80
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs CIRCUIT CONFIGURATION

CELL FAILURE RATE = 1 PER 8000 PER YEAR
BRANCH CIRCUIT = 8 P x 2400 S
MODULE = 4 x 8 FOOT (320 CELLS)

WITH NO MODULE REPLACEMENT

ONE MODULE REPLACEMENT PER CELL FAILURE

LIFE-CYCLE ENERGY COST ($/kWh)

SERIES BLOCKS PER BRANCH CIRCUIT
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs MODULE SIZE
(OPTIMUM MAINTENANCE)

CELL FAILURE RATE = 1 PER 10,000 PER YEAR
BRANCH CIRCUITS = 8 P x 2400 S, NO DIODES
CELL FILL FACTOR = 0.7
MODULE SIZE = (FT x FT)
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs MODULE SIZE
(OPTIMUM MAINTENANCE)

CELL FAILURE RATE = .0001
BRANCH CIRCUIT = 8 P X 1200, NO DIODES
FF = .70
MODULE SIZE = (FT X FT)
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs MODULE SIZE

(OPTIMUM MAINTENANCE)

CELL FAILURE RATE = .0001
BRANCH CIRCUIT = 8 P X 600's, NO DIODES
FF = .70
MODULE SIZE = (FT. X FT.)
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs SERIES/PARALLELING

(OPTIMUM MAINTENANCE)

CELL FAILURE RATE = 1 PER 10000 PER YEAR
BRANCH CIRCUIT = 2400 SERIES BY N PARALLEL
MODULE = 4 x 8 FOOT (320 CELLS)

LIFE-CYCLE ENERGY COST ($/kWh)

SERIES BLOCKS PER BRANCH CIRCUIT

4 PARALLEL

16 PARALLEL

8 PARALLEL
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs SERIES/PARALLELING
(OPTIMUM MAINTENANCE)

CELL FAILURE RATE = .0001
BRANCH CIRCUIT = 1200 SERIES X N PARALLEL
NO DIODES
FF = .70
MODULE = 4 X 4 (150 CELLS)
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs MODULE SIZE
(OPTIMUM MAINTENANCE)

CELL FAILURE RATE = .0001
BRANCH CIRCUIT = 600 SERIES X N PARALLEL
NO DIODES
FF = .70
MODULE = 4 X 4 (150 CELLS)

LIFE-CYCLE ENERGY COST ($/kWh)

SERIES BLOCKS PER BRANCH CIRCUIT
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs SERIES/PARALLELING

(OPTIMUM MAINTENANCE)

CELL FAILURE RATE = 1 PER 10000 PER YEAR
BRANCH CIRCUIT = 2400 SERIES BY N PARALLEL,
ONE DIODE PER SERIES BLOCK
MODULE = 4 x 8 FOOT (320 CELLS)

LIFE-CYCLE ENERGY COST ($/kWh)

SERIES BLOCKS PER BRANCH CIRCUIT

1 PARALLEL

4 PARALLEL

8 PARALLEL
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs SERIES/PARALLELING
(OPTIMUM MAINTENANCE)

CELL FAILURE RATE = .0001
BRANCH CIRCUIT = 1200 SERIES BY N PARALLEL
MODULE = 4 x 4 (150 CELLS)
FF = .70
1 SERIES BLOCK PER DIODE

LIFE-CYCLE ENERGY COST ($/kWh)

SERIES BLOCKS PER BRANCH CIRCUIT
LOW-COST SOLAR ARRAY PROJECT

LIFE-CYCLE ENERGY COST vs SERIES/PARALLELING
(OPTIMUM MAINTENANCE)

CELL FAILURE RATE = .0001
BRANCH CIRCUIT = 600 CELLS BY N PARALLEL
MODULE = 4 x 4 (150 CELLS)
FF = .70
1 SERIES BLOCK PER DIODE

LIFE-CYCLE ENERGY COST ($/kWh)

SERIES BLOCKS PER BRANCH CIRCUIT

SINGLE STRING
4 PARALLEL
8 PARALLEL

1 10 100 1000
LOW-COST SOLAR ARRAY PROJECT

SIMPLIFIED DESIGN OPTIMIZATION

APPROACH

- OPTIMUM = MINIMUM \( \frac{\text{LIFE-CYCLE COST}}{\text{LIFE-CYCLE ENERGY}} \)

- CONSIDER ONLY TWO REPLACEMENT STRATEGIES:
  - NO REPLACEMENT
  - MODULE REPLACED FOR EACH CELL FAILURE

- USE L-C ENERGY BASED ON 5-YEAR POWER FRACTION

- CHOOSE DESIGN AND REPLACEMENT STRATEGY GIVING LOWEST (\$/kW-h) OPTIMUM
LOW-COST SOLAR ARRAY PROJECT

OPTIMIZATION ALGORITHM

OPTIMUM = MINIMUM \left( \frac{\text{LIFE-CYCLE COST}}{\text{LIFE-CYCLE ENERGY}} \right)

= \operatorname{MINIMUM}_L \left( \frac{C_0 + \sum_{n=1}^{L} C_n (1 + k)^{-n}}{E_0 \sum_{n=1}^{L} \epsilon_n (1 + k)^{-n}} \right)

\text{THEREFORE: BALANCE INITIAL ARRAY OF PLANT + ARRAY 2 + \text{L-C O & M COST},$/kW}

\text{PLANT EFFICIENCY (100 mW/cm}^2\text{, NOCT)}

\text{ANNUAL INSOLATION (kW-h/m}^2\text{/yr)}\times\text{(L-C ENERGY FRACTION\*)}

\text{L-C ENERGY FRACTION} = \sum_{n=1}^{L} \left( \frac{\text{POWER IN YEAR } n}{\text{INITIAL POWER}} \right)(1 + k)^{-n}, \ k = \text{DISCOUNT RATE}

4/1/80
LOW-COST SOLAR ARRAY PROJECT

SIMPLIFIED DESIGN OPTIMIZATION
LIFE CYCLE EXPRESSIONS

NO MODULE REPLACEMENT:

L-C O & M COST ≈ 0 (MINIMUM UPKEEP)
L-C ENERGY FRACTION = \( \text{COMPUTED USING ARRAY DEGRADATION AT 5 YEARS} \)

ONE MODULE REPLACEMENT PER CELL FAILURE:

\[
\text{L-C O & M COST} = \left( \frac{\text{COST PER REPLACEMENT ACTION, \$}}{\text{ANNUAL CELL FAILURE RATE}} \right) \times \left( \frac{\text{CELLS PER MODULE}}{\text{MODULE AREA, m}^2} \right) \times a_{n}\]

L-C ENERGY FRACTION = \( a_{n} \)

\( a_{n} = \text{PRESENT VALUE OF UNIT ANNUITY} = \frac{1 - (1 + k)^{-n}}{k} \)
\( k = \text{DISCOUNT RATE} \)
LOW-COST SOLAR ARRAY PROJECT
VISUALIZATION OF 5 YEAR POWER FRACTION POINT

FRACTION INITIAL POWER

TIME (YEARS)

120 SERIES BLOCKS

1200

15

1

0 5 10 15 20

RR
4/1/80
## ARRAY PERFORMANCE WORKSHEET

### MECHANICAL CONFIGURATION
- **Module Size, m x m**
- **Module Area, \( A \), m\(^2\)**
- **Panel Size, m x m**
- **Cell Contact Pattern**

### ELECTRICAL CONFIGURATION
- **Total Cells per Module**
- **Series Cells per Module**
- **Parallel Cells per Module**
- **Diodes per Module**
- **Series Cells per Branch Circuit (NS)**
- **Parallel Cells per BC**
- **Series Blocks per BC**
- **Diodes per BC**
- **Series Cells per Diode**
- **Cells per Substring (MC) = NS/SB**

### SYSTEM ELECTRICAL EFFICIENCY
- **Encapsulated Cell Efficiency**
- **NOCT Efficiency**
- **Cell Mismatch Efficiency**
- **Cell Packing Efficiency**
- **Array Optical Soiling Efficiency**
- **Array Wiring Efficiency**
- **Power Conditioning Efficiency**
- **Total Plant Efficiency**

### FAILURE STATISTICS
- **Cracked Cell Density - Mfg/Shipping**
- **Failed Cell Density - Mfg/Shipping**
- **Cell Cracking Rate - Field Exposure**
- **Cell Failure Rate - Field Exposure (F)**
- **Module Yield (Mfg/Shipping)**

### ARRAY INITIAL COSTS ($/m\(^2\) of Array)
- **Module Cost before Breakage (\( C_M \))**
- **Module Yield Cost (\( C_y \)) = \( (1/yield - 1) \times C_M \)**
- **Panel Frame Structure & Assembly**
- **Panel Wiring**
- **Panel Installation**

### ARRAY REPLACEMENT COST ($/Module Failure)
- **Fault Identification**
- **Field Substitution Labor**
- **Module Repair/Replacement Labor**
- **Replacement Module Parts (\( C_M + C_y \) \times A)**

### BALANCE-OF-PLANT LIFE-CYCLE COST ($/kW)

### LIFE-CYCLE COST PERFORMANCE CALCULATION
- **\( LCOM1 = \) Initial Array Cost + LC O&M (\( \) / $/m\(^2\) )\)
- **\( LCO = \) Module Replacement Cost + Field Substitution + Cell Repair/Replacement (\( \) \)

### LIFE-CYCLE ENERGY FRACTION (\( \epsilon_{LC} \))
- For Module Replacement Each Cell Failure:
  \[ \epsilon_{LC1} = \frac{am}{(1-\epsilon_{LC})^{20} + k} \]
- For No Module Replacement:
  \[ \epsilon_{LC0} = \frac{am}{1-(1-5 \times F)} \]

### LIFE-CYCLE O&M COSTS ($/m\(^2\) of Array)
- For Module Replacement Each Cell Failure:
  \[ \frac{Cost}{Array} \times \frac{Cells}{Failure} \times \frac{am}{Rate} \times \frac{Module}{Area} \]
- For No Module Replacement:
  \[ \frac{LCOM1}{LC O&M} = \] Minor Upkeep Costs

### HOT SPOT HEATING
- **Cell Shunt Resistance**
- **Power Dissipation (cracked cell) \( P/P_{max} \)**
- **Temperature above Ambient (cracked), \( \) \( ^\circ C \)**
- **Cracked Cell Heating: OK, Marginal, Bad**
- **Open Circuit Heating: OK, Marginal, Bad**

### ANNUAL INCIDENT INSOLATION (kW-h/m\(^2\)/yr)

### COMMENTS:

7-6
LOW-COST SOLAR ARRAY PROJECT

EXAMPLE DESIGN PROBLEM

- DETERMINE OPTIMUM MODULE CONFIGURATION FOR LARGE GROUND-MOUNTED ARRAY
  - MECHANICAL CONFIGURATION/MODULAR SIZE
  - CIRCUIT DESIGN
  - MAINTENANCE/REPLACEMENT REQUIREMENTS

- ARRAY CONFIGURATION:

  ![Module](image1.png)
  ![Panel](image2.png)
  ![Array](image3.png)

MODULE

PANEL

ARRAY
LOW-COST SOLAR ARRAY PROJECT

SAMPLE PROBLEM # 1

MODULE
144 CELLS
12 PARALLEL STRINGS
12 SERIES CELLS
1 DIODE

BRANCH CIRCUIT
12 PARALLEL STRINGS
100 SERIES BLOCKS
100 MODULES
1200 SERIES CELLS

RR
4/1/80
## Mechanical Configuration
- **Module Size, m x m**: 1.2 x 1.2
- **Module Area, (A), m²**: 1.44
- **Panel Size, m x m**: 2.4 x 4.8
- **Cell Contact Pattern**: 2

## Electrical Configuration
- **Total Cells per Module**: 144
- **Series Cells per Module**: 12
- **Parallel Cells per Module**: 12
- **Series Blocks per Module**: 1
- **Diodes per Module**: 1
- **Series Cells per Branch Circuit (NS)**: 1.20
- **Parallel Cells per BC**: 12
- **Series Blocks per BC (SB)**: 100
- **Diodes per BC**: 100
- **Series Cells per Diode**: 12
- **Cells per Substring (NC)**: NS/SB

## System Electrical Efficiency
- **Encapsulated Cell Efficiency**: 0.15
- **NOCT Efficiency**: 0.92
- **Cell Mismatch Efficiency**: 0.80
- **Cell Packing Efficiency**: 0.74
- **Array Optical Soiling Efficiency**: 0.72
- **Array Wiring Efficiency**: 0.92
- **Power Conditioning Efficiency**: 0.92
- **Total Plant Efficiency**: 10.5%

## Failure Statistics
- **Cracked Cell Density: Mfg-Shipping**: 0.001
- **Failed Cell Density: Mfg-Shipping**: 0.000
- **Cell Cracking Rate: Field Exposure**: 0.000
- **Cell Failure Rate: Field Exposure (F)**: 0.999
- **Module Yield (Mfg-Shipping)**: 0.999

### Array Initial Costs ($/m² of Array)
- **Module Cost before Cell Breakage ($/m²)**: 10.5
- **Module Yield Cost ($/m²)**: 2.4
- **Panel Frame Structure & Assembly**: 1
- **Panel Wiring**: 2
- **Panel Installation**: 1
- **Installed Field Structure & Foundations**: 1
- **Land and Preparation**: 4.9
- **Total**: 10.4

### Array Replacement Cost ($/Module Failure)
- **Fault Identification**: 4
- **Field Substitution Labor**: 2.1
- **Module Repair/Replacement Labor**: 12
- **Replacement Module Parts (C_N + C_Y) x A**: 87
- **Total**: 124

### Balance-of-Plant Life-Cycle Cost ($/kW)
- **Initial Array Plant Cost $/kW**: 150
- **Array Cost $/m²**: 10.5
- **L-C O&M Efficiency x Plant Cost $/kW**: 2.28
- **Array Insolation x L-C Energy ($/kWh)**: 24.2

### Life-Cycle O&M Costs ($/m² of Array)
- **LCO1**: 0.16

### Life-Cycle Cost Performance Calculation
- **Balance Plant Cost**: (150) + (10.5)
- **L-C O&M Efficiency**: 2.28
- **Array Insolation x L-C Energy ($/kWh)**: 24.2

### Hot-Spot Heating
- **Cell Shunt Resistance**: 100
- **Power Dissipation (cracked cell) P/P_max (open circuit)**: 15
- **Temperature above Ambient (cracked), °C**: 25

### Comments:
- Cracked Cell Heating: OK, Marginal, Bad
- Open Circuit Heating: OK, Marginal, Bad
1. MECHANICAL CONFIGURATION
1.1 Module size, m x m
1.2 Module area, (A), m^2
1.3 Panel size, m x m
1.4 Cell contact pattern

2. ELECTRICAL CONFIGURATION
2.1 Total cells per module
2.2 Series cells per module
2.3 Parallel cells per module
2.4 Series blocks per module
2.5 Diodes per module
2.6 Series cells per branch circuit (NS)
2.7 Parallel cells per branch circuit
2.8 Series blocks per branch circuit (SB)
2.9 Diodes per branch circuit
2.10 Series cells per diode
2.11 Cells per substring (NC) = NS/SB

3. SYSTEM ELECTRICAL EFFICIENCY
3.1 Encapsulated cell efficiency
3.2 NOCT efficiency
3.3 Cell mismatch efficiency
3.4 Cell packing efficiency
3.5 Array optical soiling efficiency
3.6 Array wiring efficiency
3.7 Power conditioning efficiency
3.8 Total plant efficiency

4. FAILURE STATISTICS
4.1 Cracked cell density: mfg-shipping
4.2 Failed cell density: mfg-shipping
4.3 Cell cracking rate: field exposure
4.4 Cell failure rate: field exposure (F)
4.5 Module yield (mfg-shipping)

5. ARRAY INITIAL COSTS ($/m^2 of array)
5.1 Module cost before cell breakage (C_M)
5.2 Module yield cost (C_y) = (1/yield) - 1 x C_M
5.3 Panel frame structure & assembly
5.4 Panel wiring
5.5 Panel installation
5.6 Installed field structure & foundations
5.7 Land and preparation
5.8 Total

6. ARRAY REPLACEMENT COST ($/module failure)
6.1 Fault identification
6.2 Field substitution labor
6.3 Module repair-replacement labor
6.4 Replacement module parts (C_M + C_y) x A
6.5 Total

7. BALANCE-OF-PLANT LIFE-CYCLE COST ($/kW)
8. LIFE-CYCLE COST DISCOUNT RATE (k)
9. ANNUAL INCIDENT INSOLATION (kWh/m^2/yr)
10. LIFE-CYCLE ENERGY FRACTION (E_LC)
10.1 For module replacement each cell failure (E_LCI)
10.2 For no module replacement (E_LCO)
10.21 Array substring 5-year failure density
10.22 Five-year power loss fraction

11. LIFE-CYCLE O&M COSTS (LCOM)
11.1 For module replacement each cell failure (LCOMI)
11.2 For no module replacement (LCOMO)

12. LIFE-CYCLE COST PERFORMANCE CALCULATION
12.1 For module replacement each cell failure
12.2 For no module replacement

13. HOT-SPOT HEATING
13.1 Cell shunt resistance
13.2 Power dissipation
13.21 Cracked cell
13.22 Open circuit
13.3 Temperature above ambient
13.31 cracked cell
13.32 open circuit
13.4 Cracked cell heating
13.5 Open circuit heating
INSTRUCTIONS FOR PREPARING ARRAY PERFORMANCE WORKSHEET

On p. 8-3 is a reproduction of the Array Performance Worksheet. In these instructions, each line of the worksheet is indexed with a number. The instructions are indexed to correspond with the line items on the worksheet. The reference to section numbers in the notebook corresponds to numbers in the Table of Contents. The figure numbers refer to the order of the figures in the section. For example, Figure 5 of Section VI would be the fifth figure in Section VI.

1. MECHANICAL CONFIGURATION

1.1 Module Size: the length and width of the module.
1.2 Module Area: the total surface area of the module.
1.3 Panel Size: the panel size (length and width) of the array subsystem containing the module.
1.4 Cell Contact Pattern: the cell contact pattern of the cells used in the module in terms of number of contacts and orientation, taking into account the effectiveness of the contact pattern in reducing cell area loss due to cracking.

2. ELECTRICAL CONFIGURATION

2.1 Total Cells per Module: the total number of cells in a module.
2.2 Series Cells per Module: the number of cells in series between the positive and negative terminations of the module.
2.3 Parallel Cells per Module: the number of parallel strings of cells per module.
2.4 Series Blocks per Module: the number of series blocks per module.
2.5 Diodes per Module: the number of diodes per module.
2.6 Series Cells per Branch Circuit: the number of cells in series between the positive and negative terminations of a branch circuit.
2.7 Series Cells per Branch Circuit: the number of parallel strings of cells per branch circuit.
2.8 Series Blocks per Branch Circuit: the number of series blocks per branch circuit.
2.9 Diodes per Branch Circuit: the number of diodes per branch circuit.
2.10 Series Cells per Diode: the number of cells in series between the positive and negative terminations of the diode.
2.11 Cells per Substring: the number of series cells in a series block substring, i.e., between the positive and negative terminations of the series block.

3. SYSTEM ELECTRICAL EFFICIENCY

3.1 Encapsulated Cell Efficiency ($\eta_{ec}$): The encapsulated cell efficiency is the average solar cell efficiency within an
encapsulated module at Standard Test Conditions (100 mW/cm² insolation at 28°C).

\[
\eta_{ec} = \frac{\text{Module Peak Power}}{\text{No. of Cells} \times \text{Projected Cell Area} \times \text{Solar Irradiance}}
\]

The value to be used is that which is appropriate to the cell in question. For the example problem a nominal value of 0.15 is used, representing a future high-efficiency solar cell.

3.2 NOCT* Efficiency (\(\eta_{NOCT}\)): The NOCT efficiency represents the fraction of the power output measured at standard test conditions (100 mW/cm², 28°C) that is available at standard operating conditions (100 mW/cm², NOCT) when the solar cells have achieved their Nominal Operating Cell Temperature. For the purpose of the example problem a typical value of 0.92 is used, based on the typical present-day NOCT of 45°C. The value to be used for a given module may vary from this.

3.3 Cell Mismatch Efficiency, \(\eta_{MIS}\): the ratio of the efficiency of a series-parallel assembly of cells or modules measured under uniform illumination conditions to the average efficiency of the individual elements, determined from a mismatch analysis. In the absence of actual analysis, a typical value can be determined from the curves in Section II (Figs. II-4 to II-6).

3.4 Cell Packing Efficiency (\(\eta_p\)): The cell packing efficiency denotes the ratio of the total active solar cell area to the total aperture area for a flat-plate module. This term allows for area occupied by module border, bus, and interconnects. The value used must be determined for each specific module; however, for the purpose of the example problem, a typical value of 0.91 is used.

3.5 Array Optical Soiling Efficiency (\(\eta_{OS}\)): Ratio of peak power at standard reporting conditions after outdoor exposure to the peak power at standard reporting conditions at initial deployment. For the purpose of the example problem, a typical value of 0.92 is used.

3.6 Array Wiring Efficiency (\(\eta_{wire}\)): The array wiring efficiency is a term that accounts for electrical losses due to internal resistance of the wiring system. These losses are normally quite small, provided that the conductor size and material are selected on the basis of current and voltage requirements. A typical value of 0.98 is used in the example problems.

3.7 Power-Conditioning Efficiency (\(\eta_{pc}\)): The power-conditioning efficiency is a term that accounts for losses resulting from the conversion of dc voltage to ac voltage. A typical value of 0.94 is used in the example problems.

* Nominal Operating Cell Temperature
3.8 Total Plant Efficiency ($\eta_{total}$): the product of the efficiencies given in lines 3.1 to 3.7:

$$\eta_{total} = \prod \eta_i.$$ 

4. FAILURE STATISTICS

4.1 Cracked Cell Density--Mfg-Shipping: The cracked-cell density cited here is the fraction of cells encapsulated in modules that become cracked during manufacturing, shipping and field assembly of the modules. Consideration of this factor is eliminated from the example problems for the purpose of simplicity.

4.2 Failed Cell Density--Mfg-Shipping: The failed-cell density cited here is the fraction of cells encapsulated in modules that fail during manufacturing, shipping and field assembly of the modules. One method of determining this value is the statistical prediction of failures resulting from a given cracked-cell density (Line 4.1) and a given cell-contact pattern. Cracking is the major cause of cell failure. Another method is to use a mean observed value. For the purposes of the example problem a nominal value of 0.001 is used.

4.3 Cell Cracking Rate--Field Exposure: The cell-cracking rate cited here is the fraction of cells encapsulated in modules that crack during a year in the field assuming a constant cracking rate. Consideration of this factor is eliminated from the example problems for the purposes of simplicity.

4.4 Cell Failure Rate--Field Exposure (F): The cell-failure rate cited here is the fraction of cells encapsulated in modules that fail during a year in the field. One method of determining this value is the statistical prediction of failure resulting from a given cracked-cell rate (Line 4.3) and a given cell-contact pattern. Another method is to use a mean observed value. For the purpose of the example problems a nominal value of 0.0001 is used.

4.5 Module Yield--Mfg-Shipping: The module yield is given by the fraction of modules manufactured, shipped and installed in the field that produces an acceptable amount of power under given defined conditions. The value of yield is determined as described in Section III. The values used in connection with the example problems are based on a nominal cell failure density (Line 4.2) of 0.001 and an acceptable power equal to 0.90 or greater than that produced by an unfailed module.

5. ARRAY INITIAL COSTS ($$/m^2 OF ARRAY)

5.1 Module Cost Before Cell Breakage (CM): The module cost before cell breakage includes all costs associated with module manu-
facturing, shipping, and assembling in the field irrespective of the amount of power output. A typical cost of $60.00 per square meter is used in the example problems.

5.2 Module Yield Cost ($C_y$): The module yield cost is given by the following formula: $\left(\frac{1}{\text{Yield}} - 1\right) \times C_m$

where yield is determined by the method described in Paragraph 4.5 above. The value of manufacturing yield is found on Line 4.5, and that of $C_m$, the module cost, on Line 5.1.

5.3 Panel Frame Structure & Assembly: The panel frame structure and assembly cost is the total cost associated with providing a panel frame and securing the module(s) into an assembly. A typical cost of $18.00 per square meter is used in the example problems.

5.4 Panel Wiring: The panel wiring cost includes the cost of materials and labor required to interconnect the modules in the panel electrically. A typical cost of $2.00 per square meter is used in the examples.

5.5 Panel Installation: The panel installation cost includes material and labor costs associated with interconnecting the panels mechanically and electrically to the field structure and foundation to form an array. A typical cost of $1.00 per square meter is used in the example problems.

5.6 Installed Field Structure & Foundations: The installed field structure and foundations cost includes costs associated with materials, manufacture, shipping, and installation of the panel support structure and foundation. A typical cost of $18.00 per square meter is used in the example problems.

5.7 Land and Preparation: The land and preparation cost is associated with acquisition of the land, clearing, filling, etc., to make the area suitable for a photovoltaic array installation. A typical cost of $4.00 per square meter is used in the example problems.

5.8 Total: The total array initial cost is the sum of costs in Lines 5.1 to 5.7.

6. ARRAY REPLACEMENT COST

6.1 Fault Identification: The fault identification cost is associated with locating and isolating a fault (failure) in a specific module within the array. A typical cost of $4.00 per module failure is used in the example problems.

6.2 Field Substitution Labor: The field substitution labor cost is associated with removal and replacement of a module or panel causing the array performance to meet specifications. A typical cost of $21.00 per module failure is used in the example problems.
6.3 Module Repair and Replacement Labor: The module repair and replacement labor cost is associated with correcting the fault by replacing parts or otherwise repairing the module. A typical cost of $12.00 per module failure is used in the example problems.

6.4 Replacement Module Parts: The replacement module parts cost is associated with electrical and mechanical parts required to restore the module to an acceptable level of performance. A typical cost of $91.00 per module failure is used in the example problems.

6.5 Total: The total array replacement cost is the sum of costs in Lines 6.1 to 6.4.

7. BALANCE-OF-PLANT LIFE-CYCLE COST ($/kW): The balance-of-plant life-cycle costs are those system costs that are independent of array and module parameters and therefore are exclusive of the costs covered in paragraphs 5.1 to 5.7 above. They are given in terms of dollars per kilowatt of total plant output power at 100 mW/m², and NOCT conditions. For the purpose of the example problems a typical value of $150/kW is used.

8. LIFE-CYCLE COST DISCOUNT RATE (k): The life-cycle cost discount rate, k, is the discount rate used to determine the present worth of money received (energy worth) or paid out (maintenance costs) in the future. For the purpose of the example problems a typical value of 10% is used. In the analysis of a particular system the worth of capital to the particular customer using the system must be considered (e.g. utility, industrial user, homeowner, etc.)

9. ANNUAL INTEGRATED IRRADIANCE (kWh/m²/yr): The annual integrated irradiance is the average annual solar energy received at the site being considered for the system in question. For the purpose of the example problems, a typical value of 1825 kWh/m²/year is used. The value is characteristic of flat-plate arrays at the local latitude at sites receiving a moderate amount of sunlight, a value less than at sun-belt locations, such as Phoenix or Albuquerque, and greater than that at sites with a limited availability of solar energy.

10. LIFE-CYCLE ENERGY FRACTION (ε_LC): The life-cycle energy fraction is the present value of the summation of the annual fractions of initial energy output (per unit of energy). See Section VI for an explanation of this concept.

10.1 For Module Replacement Each Cell Failure (LCI): The value of ε_LC, assuming a 20-year system life and replacement of modules with each cell failure, is given by:
\[
\epsilon_{\text{LCI}} = a_\infty = \frac{[1 - (1 + k)^{-20}]}{k}
\]

where \(k\) is the discount rate (Line 8).

10.2 For No Module Replacement \((\epsilon_{\text{LCO}})\)

10.21 Array Substring 5-Year Failure Density: The array substring 5-year failure density for no module replacement is given by:

\[
1 - (1 - 5 F)^{N_C}
\]

where \(F\) equals the yearly cell failure rate (Line 4.4) and \(N_C\) is the number of cells per branch circuit substring (Line 2.11).

10.22 Five-Year Power Loss Fraction: The 5-year array power loss fraction is obtained from the appropriate curve given in Section IX based on the information given in Lines 2.7, 2.8, 2.9, cell fill factor, and the 5-year array substring failure density obtained in Line 10.21.

10.23 The life cycle energy fraction is found using this 5-year power loss fraction and Figure VII-5 in Section VII.

11. LIFE-CYCLE O&M COSTS (LCOM)

11.1 For Module Replacement Each Cell Failure (LCOMI): The life-cycle O&M costs for module replacement with each cell failure is given by:

\[
\text{LCOMI} = \frac{\text{Cost per replacement action, } S \times \text{Cell failure rate } \times \text{Cells per module } \times a_\infty}{\text{(Module Area, } m^2)}
\]

where:

- Cost per replacement action is given in Line 6.5
- Cell failure rate is given in Line 4.4
- Cells per module is given in Line 2.1
- Module area is given in Line 1.2

11.2 For No Module Replacement (LCOMO)—The life-cycle O&M costs for no module replacement is taken to be 0.
12. LIFE-CYCLE COST PERFORMANCE CALCULATION

The life-cycle cost is given by:

\[
\frac{S}{\text{kWh}} = \left( \frac{\text{Balance cost}, S/\text{kW}}{\text{cost}, S/\text{kW}} \right) + \left( \frac{\text{Initial array cost/m}^2}{\text{cost/m}^2} \right) + \left( \frac{\text{Array L-C O&M cost/m}^2}{\text{cost/m}^2} \right) + \left( \frac{\text{Plant efficiency}}{100 \text{ mW/cm}^2, \text{NOCT}} \right) \\
\left( \frac{\text{Annual insolation}}{\text{kWh/m}^2/\text{yr}} \right) \times \left( \frac{\text{L-C energy fraction}}{\text{fraction}} \right)
\]

Except for array L-C O&M cost and L-C energy fraction, the values used for the cases with module replacement and without module replacement are the same and are discussed below:

Balance-of-plant cost is given in Line 7, initial array cost is given in Line 5.8, plant efficiency is given in Line 3.8, and annual insolation is given in Line 9.

12.1 For Module Replacement Each Cell Failure: The life-cycle cost for the case of module replacement with each cell failure is determined by using the array L-C O&M cost obtained in Line 11.1 and the life-cycle energy fraction obtained in Line 10.1.

12.2 For No Module Replacement: The life-cycle cost for the case of no module replacement is determined by using the array L-C O&M cost obtained in Line 11.2 and the life-cycle energy fraction obtained in Line 10.23.

13. HOT-SPOT HEATING

13.1 Cell Shunt Resistance: The cell shunt resistance must be selected carefully with regard to the current at which it is determined, measurement at steady conditions, distribution of shunt resistances within a cell population, etc. A typical value of 100 ohms is used in the example problems.

13.2 Power Dissipation, \( P/P_{\text{max}} \)

13.21 Cracked Cell: The power dissipation in the case of a cracked cell is determined from Figure IV-4 in Section IV. The number of series blocks is equivalent to the number of series cells per diode, given in Line 2.10. The number of parallel strings is equivalent to \( 1/(\text{fraction of area lost}) \). The fraction of area lost leading to worst-case heating should be used. For the purpose of the example problems a 50% area loss was assumed.

13.22 Open Circuit: The power dissipation in the case of an open-circuit cell is determined from Figure IV-4 in Section IV. The number of series blocks is actually the number of series
blocks per diode, obtained by dividing the number of series blocks per branch circuit (Line 2.8) by the number of diodes per branch circuit (Line 2.9). The number of parallel strings is given by Line 2.7.

13.3 Temperature Above Ambient

13.31 Cracked Cell: The temperature rise above ambient for a cracked cell is obtained by taking the value of $P/P_{\text{max}}$ from Line 13.21 and using Figure IV-3 in Section IV.

13.32 Open Circuit: The temperature rise above ambient for an open-circuit cell is obtained by taking the value of $P/P_{\text{max}}$ from Line 13.22 and using Figure IV-3 in Section IV.

13.4 Cracked-Cell Heating: The criterion for classifying cracked-cell heating is based on the result obtained on Line 13.31. Values less than 50°C for the rise in cell temperature above ambient are acceptable (OK), values between 50°C and 70°C are marginal and values greater than 70°C are unacceptable (BAD).

13.5 Open Cell Heating: The criterion for classifying open-cell heating is based on the result obtained on Line 13.32. Values less than 50°C for the rise in cell temperature above ambient are acceptable (OK), values between 50°C and 70°C are marginal and values greater than 70°C are unacceptable (BAD).
## LOW-COST SOLAR ARRAY PROJECT
### NOMINAL PERFORMANCE PARAMETERS

<table>
<thead>
<tr>
<th>Module Size (ft x ft)</th>
<th>INITIAL ARRAY EFFICIENCY</th>
<th>BALANCE-OF-PLANT EFFICIENCY</th>
<th>BALANCE-OF-PLANT COSTS (1975$)</th>
<th>DISCOUNT RATE (OVER INFLATION)</th>
<th>ANNUAL INSOLATION</th>
</tr>
</thead>
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<tr>
<td></td>
<td>ENCAP. CELL EFFICIENCY</td>
<td>ELECTRICAL EFFICIENCY</td>
<td></td>
<td>10%</td>
<td>1825 kW-h/m²/yr</td>
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<tr>
<td></td>
<td>NOCT EFFICIENCY</td>
<td>MODULE SOILING EFFICIENCY</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PACKING EFFICIENCY</td>
<td>BALANCE-OF-PLANT EFFICIENCY</td>
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<td></td>
</tr>
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<td></td>
<td>ARRAY EFFICIENCY SUBTOTAL</td>
<td>BALANCE-OF-PLANT SUBTOTAL</td>
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<td></td>
</tr>
<tr>
<td>2 x 4</td>
<td>0.15</td>
<td>0.92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 x 4</td>
<td>0.15</td>
<td>0.92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 x 8</td>
<td>0.15</td>
<td>0.92</td>
<td>150 $/kW</td>
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<tr>
<td></td>
<td>0.123</td>
<td>0.126</td>
<td>0.85</td>
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<tr>
<td></td>
<td>0.128</td>
<td>0.85</td>
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</table>
### LOW-COST SOLAR ARRAY PROJECT

#### NOMINAL ARRAY COSTS

(1975 $)

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>UNITS</th>
<th>MODULE SIZE (ft x ft)</th>
<th>2 x 4</th>
<th>4 x 4</th>
<th>4 x 8</th>
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<tr>
<td>INITIAL:</td>
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</tr>
<tr>
<td>MODULE DIRECT COST</td>
<td>$/m²</td>
<td>60</td>
<td>60</td>
<td>60</td>
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<tr>
<td>MODULE YIELD COST*</td>
<td>$/m²</td>
<td>0-5</td>
<td>0-8</td>
<td>0-23</td>
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<tr>
<td>• MODULE SUBTOTAL</td>
<td></td>
<td>60-65</td>
<td>60-68</td>
<td>60-83</td>
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<tr>
<td>PANEL FRAME</td>
<td>$/m²</td>
<td>24</td>
<td>18</td>
<td>15</td>
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<tr>
<td>PANEL WIRING</td>
<td>$/m²</td>
<td>2-4</td>
<td>2-3</td>
<td>1-2</td>
<td></td>
</tr>
<tr>
<td>• PANEL SUBTOTAL</td>
<td></td>
<td>26-28</td>
<td>20-21</td>
<td>16-17</td>
<td></td>
</tr>
<tr>
<td>PANEL INSTALLATION</td>
<td>$/m²</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>INSTALLED ARRAY STRUCT</td>
<td>$/m²</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>• ARRAY TOTAL</td>
<td></td>
<td>109-116</td>
<td>103-112</td>
<td>99-123</td>
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<tr>
<td>PER REPLACEMENT ACTION:</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>FAULT IDENTIFICATION</td>
<td>$/PANEL</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>PANEL SUBSTITUTION LABOR</td>
<td>$/PANEL</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>MODULE REPLACEMENT LABOR</td>
<td>$/MOD</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>REPLACEMENT MODULE PARTS (INC 1% INVENTORY COST)</td>
<td>$/m²</td>
<td>61-66</td>
<td>61-69</td>
<td>61-84</td>
<td></td>
</tr>
</tbody>
</table>

*1 CELL FAILURE PER 1000 DURING ASSEMBLY/SHIPPING/INSTALLATION*
LOW-COST SOLAR ARRAY PROJECT

MANUFACTURING YIELD CALCULATION

\[ f(x) = \frac{e^{-\mu x}}{x!} \]

\[ \mu = np = 144 \times 0.001 = 0.144 \]

\begin{center}
\begin{tabular}{|c|c|c|c|}
\hline
\( x \) & 1 & 2 & 3 \\
\hline
\( f(x) \) & 0.125 & 0.00896 & 0.00007 \\
\hline
\end{tabular}
\end{center}

SINCE TWO OR MORE FAILED CELLS
CAUSE POWER LOSS GREATER THAN 0.1,
\[ MY = 1.0 - (0.00896 + 0.00007) = 0.991 \]
LOW-COST SOLAR ARRAY PROJECT

MODULE POWER LOSS VERSUS MODULE SERIES/PARALLELING

(1 TO 3 FAILED CELLS PER MODULE)

1 PARALLEL *

2 PARALLEL *

HOT SPOT PROBLEM

4 PARALLEL *

CELL FAILURES PER MODULE

1, 2, 4 PARALLEL STRINGS BASED ON 1 CELL FAILURE ONLY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS FRACTION CALCULATION

- INTERPOLATE BETWEEN 8 AND 16 PARALLEL STRINGS ARRAY

POWER LOSS CURVES

- USE YEARLY FAILURE RATE OF 0.0001. AFTER 5 YEARS,

FAILURE RATE = 0.0005

\[ F_{SS} = 1 - (1 - F)_{NC} \]

\[ F_{C} = 0.0005 \]

\[ N_{C} = 12 \]

\[ F_{SS} = 0.00598 \]

CCG
3/31/80
LOW-COST SOLAR ARRAY PROJECT
ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.70
1 SERIES BLOCK PER DIODE

SUBSTRING FAILURE DENSITY

F_SS, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

16 PARALLEL STRINGS
FF = 0.70
1 SERIES BLOCK PER DIODE

ARRAY POWER LOSS FRACTION

Fss, SUBSTRING FAILURE DENSITY

SERIES BLOCKS PER BRANCH CIRCUIT

2400
1000
500
250
100
50
25
12
6
2
1

CCG
3-31-80
LOW-COST SOLAR ARRAY PROJECT
LIFE-CYCLE ENERGY FRACTION vs ARRAY POWER FRACTION AT 5 YEARS

20-YEAR LIFE-CYCLE ENERGY FRACTION

5-YEAR POWER FRACTION ($P_5/P_0$)

DISCOUNT RATE %
LOW-COST SOLAR ARRAY PROJECT

EFFECT OF SERIES/PARALLELING ON HOT SPOT CELL HEATING

(SHUNT RESISTANCE = 100Ω)

(SHUNT RESISTANCE = 10Ω)

SERIES BLOCKS

PARALLEL STRINGS

POWER DISSIPATED (P/P$_{\text{MAX}}$)

PARALLEL STRINGS

CCG
12/6/79
LOW-COST SOLAR ARRAY PROJECT

TYPICAL HOT-SPOT HEATING LEVEL FOR FLAT-PLATE MODULE

HOT-SPOT POWER DISSIPATION ($P/P_{\text{MAX}}$)

MEASURED DATA FOR GLASS MODULE WITH 4 in. CELLS

IRRADIANCE ONLY ($\alpha = 0.85$)

IRRADIANCE ONLY + ELECTRICAL POWER
LOW-COST SOLAR ARRAY PROJECT
SAMPLE PROBLEM # 2

MODULE
144 CELLS
6 PARALLEL STRINGS
24 SERIES CELLS
2 SERIES BLOCKS
1 DIODE

BRANCH CIRCUIT
6 PARALLEL STRINGS
100 SERIES BLOCKS
50 MODULES
1200 SERIES CELLS
ARRAY PERFORMANCE WORKSHEET

MECHANICAL CONFIGURATION
Module Size, m x m = 1.2 x 1.2
Module Area, (A), m² = 1.44
Panel Size, m x m = 2.4 x 4.8
Cell Contact Pattern = -

ELECTRICAL CONFIGURATION
Total Cells per Module = 144
Series Cells per Module = 24
Parallel Cells per Module = 6
Series Blocks per Module = 2
Diodes per Module = 1
Series Cells per Branch Circuit (NS) = 1200
Parallel Cells per BC = 6
Series Blocks per BC (SB) = 100
Diodes per BC = 50
Series Cells per Diode = 24
Cells per Substring (NC) = NS/SB = 12

SYSTEM ELECTRICAL EFFICIENCY
Encapsulated Cell Efficiency = 0.15
NOCT Efficiency = 0.92
Cell Mismatch Efficiency = 0.98
Cell Packing Efficiency = 0.97
Array Optical Soiling Efficiency = 0.92
Array Wiring Efficiency = 0.93
Power Conditioning Efficiency = 0.94
Total Plant Efficiency = 0.73

FAILURE STATISTICS
Cracked Cell Density: Mfg-Shipping = -
Failed Cell Density: Mfg-Shipping = 0.001
Cell Cracking Rate: Field Exposure = -
Cell Failure Rate: Field Exposure (F) = 0.0001
Module Yield (Mfg-Shipping) = 0.8369

ARRAY INITIAL COSTS ($/m² of Array)
Module Cost before Cell Breakage (Cₜ) = 60
Module Yield Cost (Cₜ) = (1/yield - 1) x Cₜ = 2.28
Panel Frame Structure & Assembly = 18
Panel Wiring = 2
Panel Installation = 1
Installed Field Structure & Foundations = 18
Land and Preparation = 4
Total = 112

ARRAY REPLACEMENT COST ($/Module Failure)
Fault Identification = 4
Field Substitution Labor = 21
Module Repair/Replacement Labor = 12
Replacement Module Parts (Cₜ + Cₕ) x A = 100
Total = 137

BALANCE-OF-PLANT LIFE-CYCLE COST ($/kWh)
LIFE-CYCLE COST DISCOUNT RATE (k) = 10%
ANNUAL INTEGRATED IRRADIANCE (kWh/m²/yr) = 1825

LIFE-CYCLE ENERGY FRACTION (εₐₖ)
- For Module Replacement Each Cell Failure:
εₐₖ = [1 - (1 + k)-20]/k = 8.51
- For No Module Replacement:
5-year Power Loss Fraction (from plot) = 0.03

LIFE-CYCLE O&M COSTS ($/m² of Array)
- For Module Replacement Each Cell Failure:
εₐₖ = [137 x 0.001 x 144 x 2.51 / 1.44] = 11.66
- For No Module Replacement:
LCO₂ = Minor Upkeep Cost = 0

LIFE-CYCLE COST PERFORMANCE CALCULATION
εₐₖ = (Balance of Plant Cost, $/kWh) x (Initial Array Cost) x (Array L-C O&M Cost/kWh) x (Plant Efficiency)

HOT-SPOT HEATING

Cracked Cell Heating: OK, Marginal, Bad
Open Circuit Heating: OK, Marginal, Bad

COMMENTS:
Array Power Loss Fraction

This section provides a set of array power loss fraction curves, indexed according to:

(1) The number of parallel strings per branch circuit;

(2) The cell fill factor; and

(3) The number of series blocks per diode.

<table>
<thead>
<tr>
<th>No. of Parallel Strings</th>
<th>Fill Factor</th>
<th>No. of Series Blocks per Diode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.70</td>
<td>1</td>
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<tr>
<td>4</td>
<td>.70</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>.76</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>.70 or .76</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>.70 or .76</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>.70 or .76</td>
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</tr>
<tr>
<td>16</td>
<td>.76</td>
<td>12</td>
</tr>
</tbody>
</table>
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

SINGLE STRING
FF = .70
ONE SERIES BLOCK PER DIODE

ARRAY POWER LOSS FRACTION

F_{ss}', SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

4 PARALLEL STRINGS
FF = 0.70
NO DIODES

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY

SERIES BLOCKS
PER BRANCH CIRCUIT

2400
1000
500
250
100
50
25
12
6
2
1
LOW-COST SOLAR ARRAY PROJECT
ARRAY POWER LOSS

4 PARALLEL STRINGS
FF = 0.76
NO DIODES

F_{SS}, SUBSTRING FAILURE DENSITY

ARRAY POWER LOSS FRACTION

SERIES BLOCKS PER BRANCH CIRCUIT

2400
1000
500
250
100
50
25
12
6
2
1
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

4 PARALLEL STRINGS
FF = 0.70 OR 0.76
1 SERIES BLOCK PER DIODE

FSS, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

4 PARALLEL STRINGS
FF = 0.70 or 0.76
4 SERIES BLOCKS PER DIODE

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY

CCG
3-31-80
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

4 PARALLEL STRINGS
FF = 0.70 or 0.76
8 SERIES BLOCKS PER DIODE

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY

SERIES BLOCKS PER BRANCH CIRCUIT

0.0001 0.001 0.01 0.1 1.0

0.0001 0.001 0.01 0.1 1.0

0.01 0.1

250 500
LOW-COST SOLAR ARRAY PROJECT
ARRAY POWER LOSS

4 PARALLEL STRINGS
FF = 0.70 or 0.76
12 SERIES BLOCKS PER DIODE

ARRAY POWER LOSS FRACTION

SERIES BLOCKS PER BRANCH CIRCUIT

F_{SS}, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.70
NO DIODES

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY

SERIES BLOCKS PER BRANCH CIRCUIT

2400
1000
500
250
100
50
25
12
6
2
1
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.70
1 SERIES BLOCK PER DIODE

FSS, SUBSTRING FAILURE DENSITY

 SERIES BLOCKS PER BRANCH CIRCUIT

2400
1000,500
250
100
50
25
12
6
2
1

0.0001 0.001 0.01 0.1 1.0

0.0001 0.001 0.01 0.1 1.0

CCG
3-31-80
LOW-COST SOLAR ARRAY PROJECT
ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.70
4 SERIES BLOCKS PER DIODE

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY

SERIES BLOCKS PER BRANCH CIRCUIT

1 2 6 12 25 50 100 250 500 1000 2400
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.70
8 SERIES BLOCKS PER DIODE

ARRAY POWER LOSS FRACTION

F_{SS}, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT
ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.70
12 SERIES BLOCKS PER DIODE

FSS, SUBSTRING FAILURE DENSITY

ARRAY POWER LOSS FRACTION

SERIES BLOCKS PER BRANCH CIRCUIT
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.76
NO DIODES

ARRAY POWER LOSS FRACTION

F_{ss}, SUBSTRING FAILURE DENSITY

CCG
12/6/79
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.76
1 SERIES BLOCK PER DIODE

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT
ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.76
4 SERIES BLOCKS PER DIODE

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY

SERIES BLOCKS PER BRANCH CIRCUIT

100
50
25
12
6
2
1

100 - 2400

0.0001 0.001 0.01 0.1 1.0

0.0001

0.001

0.01

0.1

1.0

25
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.76
8 SERIES BLOCKS PER DIODE

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

8 PARALLEL STRINGS
FF = 0.76
12 SERIES BLOCKS PER DIODE

FSS, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT
ARRAY POWER LOSS

16 PARALLEL STRINGS
FF = 0.70
NO DIODES

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

16 PARALLEL STRINGS
FF = 0.70
1 SERIES BLOCK PER DIODE

FSS, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

16 PARALLEL STRINGS
FF = 0.76
NO DIODES
LOW-COST SOLAR ARRAY PROJECT
ARRAY POWER LOSS

16 STRINGS IN PARALLEL
FF = 0.76
1 SERIES BLOCK PER DIODE

F_SS, SUBSTRING FAILURE DENSITY

ARRAY POWER LOSS FRACTION

SERIES BLOCKS PER BRANCH CIRCUIT
1 2 6 12 25 50 100 250
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

16 PARALLEL STRINGS
FF = 0.76
4 SERIES BLOCKS PER DIODE

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

16 PARALLEL STRINGS
FF = 0.76
8 SERIES BLOCKS PER DIODE

FSS, SUBSTRING FAILURE DENSITY
LOW-COST SOLAR ARRAY PROJECT

ARRAY POWER LOSS

16 PARALLEL STRINGS
FF = 0.76
12 SERIES BLOCKS PER DIODE

ARRAY POWER LOSS FRACTION

FSS, SUBSTRING FAILURE DENSITY
ABSTRACT

As part of the Jet Propulsion Laboratory's Low-Cost Solar Array Project, a program of module and array circuit-design studies has been carried out to improve the reliability of modules and to develop strategies for improving array system fault tolerance. Analyses to determine optimum element configuration and circuit design strategies, made with the use of a computer model that uses the measurable characteristics of cells and modules as input, are described and results are presented.

Guidelines for implementing strategies to deal with specific module and array system circuit-design problems, involving use of series-parallel and bypass diodes to reduce module and array system power losses resulting from anomalies, are offered. Several different power-loss conditions resulting from anomalies are considered.

INTRODUCTION

Certain faults occur in terrestrial photovoltaic modules, both at beginning of life and throughout field experience. These faults occur as a result of initial mismatch of photovoltaic cell characteristics, and of environmentally induced stress or stress induced during maintenance in the field. In addition to mismatch, causes include open-circuit cell interconnects and cracked cells. One failure mechanism that can cause severe cell cracking is hail impact. Cell failure can also be caused by uneven shading of cells.

The objective of the task described here is to investigate the effectiveness of certain circuit-design strategies in ameliorating the effects of faults on modules and on array-system performance. The task has yielded a set of guidelines for incorporating appropriate strategies into module and array-system circuit designs. The objective of the guidelines is to maximize the reliability of modules and arrays by developing fault-tolerant circuitry.

Nomenclature

Definitions and nomenclature to be used in the discussion are presented in Figure 1. A photovoltaic module consists of solar cells electrically connected in series and/or parallel. A string of cells connected in a pure series arrangement is referred to as a series string or substring. When series strings are wired in a parallel configuration, they are called parallel strings and a group of parallel strings connected at their end points is called a series block. Modules are grouped together in panels for structural purposes, forming an array. A branch circuit is composed of a group of series blocks between the positive and negative termination points of a power conditioner. Therefore, an array is a collection of branch circuits. The series blocks may be whole modules or parts of modules. In the former case, series blocks are connected together through connectors external to the modules; in the latter, some series blocks are connected by internal module wirings, while groups of series blocks are connected externally. No distinction is made in the type of connections involved in this discussion.

Circuit Design Strategies

Two important circuit design strategies are

Figure 1. Series-Parallel Nomenclature

Panel 1. Series-Parallel Nomenclature

A-1
considered in the task. The use of increasing numbers of series blocks and parallel strings per module and branch circuit as a circuit-design strategy, called series-parallelizing, is accomplished by making cross-connections between parallel strings of cells after selection of the appropriate number of parallel strings. These cross-connections are also called cross-ties.

The second circuit-design strategy considered is the use of bypass diodes. A bypass diode is connected in parallel with a given number of series blocks. The bypass diode does not conduct unless the voltage across the series block is reversed from normal polarity by a small amount, usually 0.5 V. Then the diode passes virtually unlimited current at its operating voltage in the range of interest of this study. The purpose of using a diode is to prevent the series block in question from operating at a high reverse voltage: in this state it absorbs power from the system, and cells operating with reversed polarity become hot, leading to physical degradation of cell and module. This condition is called back-biasing; its result is termed the hot-spot problem, from the fact that cells do not heat uniformly, but develop local hot-spots.

A series block or individual cell can become back-biased from several causes. The open-circuiting of one or more cells or series strings in a series block can cause the series block to operate at a current level that reverses its voltage. The same thing happens to a cell when it cracks, taking active surface area from the circuit. In fact, a cell can be considered as a sum of parallel elements: loss of one element is analogous to loss of a cell string in a series block.

Another cause of back-biasing is the non-uniform shading of cells or series blocks. Those that are shaded to a greater degree than the rest of a module or branch circuit will be back-biased.

Each circuit design strategy has advantages and disadvantages. Two of the factors that must be considered are:

(1) Cost of implementing the technique versus gain in module performance and reliability (1).

(2) Gain in the area of concern, such as power, may be vitiated by worsening of the situation in another area, such as hot-spot heating.

ANALYTICAL APPROACH

The approach used in the task involves the use of a computer model with appropriate statistical analyses. The computer model calculates cell and module I-V curves based on the following equation:

\[
I = I_{sc} \left( 1 - e^{\left(\frac{2.76}{V_{oc}} - 1\right)} \right) \frac{V}{R_{sh}}
\]

where:

- \( I \) = current of element at voltage \( V \)
- \( I_{sc} \) = short-circuit current
- \( FF \) = fill factor
- \( V_{oc} \) = open-circuit voltage
- \( R_{sh} \) = shunt resistance

The photovoltaic parameters are those commonly measured for cells and modules.

The computer model simulates various circuit designs with and without bypass diodes by adding the I-V curves of the component elements. An element is any cell or combination of cells located in a module or branch circuit. Failures can be introduced by adding I-V curves of the elements containing the types of failures being considered.

Because of the many elements involved in the typical multi-array system a statistical approach must be taken. There are two aspects to the requirements for a statistical treatment. First, a group of solar cells will have a statistical distribution of the characteristic parameters. The computer can select values of cell characteristic parameters from a given statistical distribution and assign them to the cell as it combines I-V curves.

Second, the locations of multiple failures are significant when the number of failures is not large enough to saturate the array system. Thus, the most likely location of multiple failures has an important effect on array performance. The statistical treatment of failure location has been performed in conjunction with the computer model but not internal to it. The output of the statistical model is used to define the number of cells most likely to fail for a given wiring configuration.

The output of the circuit-design computer model provides the I-V curve of the resultant combination of elements along with the maximum power of the combination. These results are combined with a statistical evaluation of the state of a given photovoltaic system to determine the most probable power output under given failure rates.

MODULE AND SYSTEM PERFORMANCE CONSIDERATIONS

Several different aspects of the performance of modules and arrays are considered in the following discussion of the methods used to determine the optimum circuit-design strategies to improve performance.
Mismatch Losses

The statistical distribution of cell parameters leads to electrical mismatch losses in modules. This occurs when cells (or series blocks) whose short-circuit currents vary considerably are combined.

The effect of mismatch is to limit the short-circuit current of the combination to a value closer to the low end of the distribution than to the average. This means that power available from the combination will be less than the sum of power of the individual cells. This is called a mismatch loss. Similar results would occur with elements combined in parallel based on open-circuit voltage. In practice these losses are not significant for a histogram distribution used is also given. Comparison of Mismatch Losses occur with elements combined in parallel based on open-circuit voltage. In practice these losses are not significant. Mismatch in fill factor is not important, since the mismatch averages out, producing no power loss.

A Monte Carlo analysis using the computer model, with Isc values selected from a histogram distribution, was used to simulate mismatch losses in modules occurring before any field degradation. The only strategy relevant to this case is series-parallel. Figure 2 is an example of mismatch loss as a function of the number of parallel strings and series blocks in the module. The short-circuit current distribution used is also given. Comparison of similar results indicates that mismatch losses are not significant for a histogram distribution having a half-width of 5% to 10%. As the short-circuit variation becomes more pronounced, losses become more significant. In this case increasing the number of parallel strings and series blocks in the module provides reduction of losses.

Figure 2. Module Mismatch as a Function of Series-parallel for a Given Short Circuit Current Distribution

Manufacturing Yield

Another aspect of module performance is manufacturing yield, which is the fraction of modules coming off the assembly line and/or delivered at the site that are acceptable for use in the field. The yield is based on the amount of power deviation from an established average that is considered acceptable for a module. One way to increase yield is the use of series-parallel, which was used in the analyses discussed here. Bypass diodes could also be used, and are discussed below in connection with array power loss.

Figure 3 gives a comparison of the values of module manufacturing yield obtained for different numbers of parallel strings and series blocks. In this case, a module cell failure rate of 1/1000 and an acceptable power deviation from average of 10% were assumed. As can be seen from the figure, series-parallel has a significant effect.

Fault Tolerance

Another module and array system performance criterion is fault tolerance. Faults arise primarily from cell failures that, in turn, arise from cracking. These faults lead to array power degradation and heating problems.

The use of series-parallel and bypass diodes is important in enhancing fault tolerance. To ascertain the effectiveness of various degrees of series-parallel, the power output of an array under specific failure conditions must be determined for a variety of series-parallel configurations with varying numbers of diodes. Power loss varies inversely with degree of series-parallel, but limits imposed by physical constraints, as well as those of cost and of thermal problems, make it impossible to achieve negligible power loss by means of unlimited series-parallel. Thus a
number of configurations must be considered, and those offering acceptable power loss must be investigated in the light of the system constraints.

In the analysis described below, only open-circuit failures were considered. The analysis of the effects of failures on a large array system requires a complex statistical approach; in order to simplify it, only the statistics of the occurrence of open failures were considered. With the number of cells per substring decreasing to approach one, the results of an analysis for open cells become applicable to an analysis of partially failed cells. For example, a series block of eight parallel substrings with one failed open can be treated as a single out of four parallel substrings and one cell 50% degraded. The consideration of only open failures allows the determination of branch-circuit power loss as a function of substring failure density independently of the number of cells per substring. In order to relate the results to a given array system, the appropriate cell failure density must be related to substring failure density.

Fault tolerance optimization at the module level is a relatively simple matter because modules cannot tolerate more than one or two failures without losing significant power. Analysis of an array system is far more complicated since it can contain many failures. The difficulty arises in locating the failures. Their locations can be defined in terms of the types of elements created; thus, a branch circuit can be described as containing Nc cells per substring, Ns series blocks, and Np parallel strings. Assuming that a branch circuit contains three open cells, several possibilities exist. One of these is the occurrence of all three failures in separate series blocks. In this case the branch circuit would consist of Ns-3 normal series blocks and three series blocks with one of Np parallel strings failed. Table 1 considers the spectrum of possible cases with three failures.

The probability of occurrence of each state of the array in Table 1 can be ascertained from a given distribution. The I-V curves of the various states can be simulated by the computer model.

**Determination of Array Power Degradation**

A method, developed to determine array power degradation, is described here. This method is meant to serve as a tool for array circuit designers to use in optimization studies.

The method applicable to various systems, must be used subject to a number of module and array constraints, leading to the requirement for parametric analyses. The parameters of concern include: module and branch circuit voltage level (cells per substring); cell fill factor; degree of series-parallelizing; number of bypass diodes used, and level of cell failures.

<p>| Table 1. Possible Combinations of Series Blocks with a Given Number of Failed Substrings for Three Failed Cells |
|-------------------------------------------------|-------------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Failure Location</th>
<th>No. of Normal Series Blocks</th>
<th>No. of Failed Series Blocks with x Failed Substrings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same series block</td>
<td>Ns</td>
<td>3</td>
</tr>
<tr>
<td>3 separate substrings</td>
<td>Ns</td>
<td>1</td>
</tr>
<tr>
<td>2 separate substrings</td>
<td>Ns-1</td>
<td>1</td>
</tr>
<tr>
<td>1 separate substring</td>
<td>Ns-2</td>
<td>1</td>
</tr>
</tbody>
</table>

**Objective of Method**

The end result of applying this method is the generation of a family of curves giving array power degradation as a function of the number of series blocks and parallel strings per branch circuit. Power degradation is plotted against the substring failure density. An example of this is shown in Figures 4 and 5 for the case of eight parallel strings. The two figures allow a comparison of the results obtained with and without bypass diodes. Similar results have been generated for branch circuits with one, four and 16 parallel strings and for different diode placement.

Substring failure density was chosen as a determining parameter instead of cell failure density in order to limit the number of curves required. If cell failure density had been used, instead of one set of curves for each case illustrated by Figures 4 or 5, a separate set would have had to be developed for each branch circuit with a different number of cells per substring.

The appropriate substring failure density to be used, for a given array, in conjunction with Figures 4 or 5 is determined from the following equation:

\[
F_{SS} = 1 - (1 - F_c)N_c \tag{2}
\]
where:

- \( F_{ss} \) = substring failure density
- \( F_c \) = cell failure density
- \( N_c \) = number of cells per substring

The binomial distribution, which is given below, is used to determine the appropriate probabilities.

\[
f(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}
\]

where \( f(x) \) is the probability of the occurrence of \( x \) events in \( n \) trials, if the probability of occurrence of the event in any given trial is \( p \) and \( Q = 1 - P \), the probability of the event not occurring.

The degree of failure associated with the branch circuits in an array is the first determination made. This is measured by the number of failed substrings, \( x \), in a given branch circuit, and in turn, by the fraction of series blocks, \( F_x \), which have this number of failed substrings. The fraction of series blocks having different numbers of failed substrings is determined as a function of substring failure density. An example of this is shown in figure 6.

The power loss associated with branch circuits having a given degree of failure is calculated. A starting point is the calculation of the power loss of a branch circuit where all failed series blocks have the same number of failed substrings. Figure 7 gives the branch circuit power loss for eight parallel strings as a function of the number of series blocks having 1, 2, ..., 7 failed substrings.

The branch circuit power loss for combinations of failed substrings at a given substring failure density is then determined. An example of the procedure follows. Consider a branch circuit having a fraction, \( F_x \), of series blocks with \( x \) failed substrings and \( F_{x-1} \) with \( x-1 \) failed substrings. Assume \( F_x \ll F_{x-1} \). (If \( F_x \approx F_{x-1} \), the series blocks with \( x \) failed substrings dominate and power loss can be computed based on them.) This situation will lead to one of two possible cases. The first is the case in which the \( F_x \) series blocks dominate (similar to the case when \( F_x \approx F_{x-1} \)). The more difficult case to analyze occurs when the power loss due to the \( F_x \) series blocks is less than or equivalent (or slightly greater) to that with \( F_{x-1} \) series blocks. Since the values of \( F_x \) and \( F_{x-1} \) are based on a generalized case, caution must be used in specific cases when determining these fractions.

Consider the example of a branch circuit having 2000 series blocks with \( F_x = .005 \) and \( F_{x-1} = .065 \). This branch circuit would have 10 series blocks with \( x \) failed substrings and 130 with \( x-1 \) failed substrings. The branch circuit power loss would be computed for 130 series blocks with \( x-1 \) failed substrings. A second value would be computed for 10 series blocks with \( x \) failed substrings, assuming that there are also 130 series blocks with \( x-1 \) failed substrings. This second power loss must be estimated for combinations of \( x \) and \( x-1 \) failed substrings. The curves used in doing this are similar to those shown in Figure 7.

**Procedural Steps**

The use of the method involves a set of procedures which are a combination of statistical analyses and use of the computer model which combines I-V curves of failed and unfailed elements.
The array power loss is determined by summing the power losses for branch circuits having one failed substring, a maximum of two failed substrings, and so on. In the summation each power loss is weighted by the fraction of branch circuits having series blocks (at least one) with a maximum of the corresponding number of failed substrings. In other words, this fraction of branch circuits will have series blocks with numbers of failed substrings up to and including the maximum.

Results of Method

The results, illustrated in Figures 4 and 5, can be used to develop guidelines for module and array circuit design. The actual curves used depend on the module and array design parameters, cited above, which act as constraints. These constraints will then define the minimum and maximum series-parallel configurations allowed, which in turn determine which and how many sets of curves are required. A cell-failure rate must be determined either based on experience with similar systems or on the most likely value. Next those series-parallel configurations and diode placement which lead to acceptable system performance in terms of power degradation must be identified. Finally, those that are unacceptable from a hot-spot standpoint must be discarded as discussed below.

If a cell failure rate per year is defined, a plot of array power loss versus time can be made. Figures 8 and 9 provide a history of power loss over 20 years for an annual cell failure rate of one per 10,000. Shown for comparison, for the case of eight parallel strings, is the effect of using diodes which leads to a decrease in power degradation.

Curves such as those shown in Figures 8 and 9 facilitate identification of optimum circuit configurations based on operation over a given number of years. It also allows the option of determining a failure replacement policy and...
Figure 9. Array Power Degradation vs Time for a Constant Yearly Cell Failure Rate of 1/10,000, for Eight Parallel Strings With Bypass Diodes, for Cells Having a .70 Fill Factor

Figure 10. Visualization of Hot-Spot Cell Heating

Although increasing the number of series blocks decreases power loss, this strategy by itself increases the hot-spot problem. The severity of the problem depends on certain cell characteristics and on the operating point of the system under question.

A simple technique can be used to determine the upper limit on the amount of energy absorbed by a group of cells in a back-bias condition. The circuit element in question can be considered as being composed basically of two parts. One part contains failed cells, either open failures or partial cell failures (cracked cells); the second part is composed of unfailed cells. Each part is composed of one or more series blocks. Figure 10 demonstrates the technique, applied with the entire element operating at short-circuit current conditions. This represents the worst-case field condition of a branch circuit being short-circuited for safety during maintenance, or a portion of a branch circuit inside a bypass diode. Since the entire element is operating at or near short-circuit conditions, the back-biased portion must be operating at a negative voltage whose magnitude is equal to that of the positive voltage of the unfailed portion. The exact operating voltage can be found by reflecting the I-V curve of the unfailed portion into the negative quadrant and determining the intersection of the two curves. This provides the voltage of the failed element. The product of this voltage and the operating current (essentially the short circuit current of the total element) gives the power absorbed by the failed element. That power drain will be distributed among all the cells carrying current.

A useful tool can be developed by drawing a family of curves representing the I-V curves of the failed and unfailed elements discussed above. The curves represent elements containing different numbers of parallel strings and series blocks as indicated in Figure 11. The abscissa is calibrated in terms of power absorbed per cell as a ratio of this power to the maximum power produced by an unfailed cell. Therefore, Figure 11 can be used to determine the power absorbed by the remaining cells in a series block of n substrings when one substring becomes open circuited. The intersection of the n parallel horizontal line with that vertical line representing the total number of series blocks involved gives the appropriate power ratio. If cracked cells are used, an approximate answer can be found by considering a cracked cell to be composed of parallel elements.

The slope of the horizontal lines is a critical factor (Figure 11). This slope is determined by the value of the shunt resistance of the cells in question. The curves shown in the figure were determined for a nominal value of 100 ohms. A change in this value of an order of magnitude is required to alter the slope appreciably. A larger value would flatten out the curve. Therefore, the most representative value of shunt resistance should be used in determining the existence of a hot-spot problem.

The most useful design strategy for reducing hot-spot problems is the use of bypass diodes. The hot-spot problem increases as the number of series blocks increases (Figure 11). Decreasing the number of series blocks in a branch circuit may be physically impossible; the smallest series block may be a module whose size is fixed. However, a bypass diode will remove all the series blocks in parallel with it if a back-bias situation occurs. It effectively reduces the number of series blocks of concern in Figure 11 to those in parallel with it. In addition, use of bypass diodes allows the acceptable number of series blocks to be increased for reduced power loss or other considerations. Adding diodes, when
physically and economically feasible, can only increase the fault tolerance of an array system.

CONCLUSIONS

Techniques have been described for evaluating the effects of different circuit design strategies on the fault tolerance of large terrestrial photovoltaic arrays. This has been accomplished by developing a generalized set of curves which can be adapted to a variety of array systems. The techniques required to generate the curves are somewhat complex but can be easily duplicated with the aid of a computer. The results obtained allow computation of the array power degradation as a function of cell failure rate for various circuit design configurations. It is also possible to find out if back-biasing occurring under a given configuration will lead to hot-spot problems.

As a result of analyses performed in the task some general guidelines can be given relative to circuit design strategies. Specific conclusions for a particular array can only be drawn after a careful analysis using the characteristic cell, module and array parameters. The general conclusions include the following:

(1) The use of bypass diodes is the best circuit design tool to reduce power loss and hot-spot problems.

(2) The paralleling of cell strings within modules is effective at reducing cell mismatch losses and increasing manufacturing yield.

(3) The use of increased number of series blocks, although leading to reduced power loss, can exacerbate hot-spot problems and should be accompanied by the use of bypass diodes.

REFERENCES

PHOTOVOLTAIC DESIGN OPTIMIZATION FOR TERRESTRIAL APPLICATIONS *

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ABSTRACT

As part of the Jet Propulsion Laboratory's Low-Cost Solar Array Project, a comprehensive program of module cost-optimization has been carried out. The objective of these studies has been to define means of reducing the cost and improving the utility and reliability of photovoltaic modules for the broad spectrum of terrestrial applications.

This paper describes one of the methods being used for module optimization, including the derivation of specific equations which allow the optimization of various module design features. The method is based on minimizing the life-cycle cost of energy for the complete system. Comparison of the life-cycle energy cost with the marginal cost of energy each year allows the logical plant lifetime to be determined. The equations derived allow the explicit inclusion of design parameters such as tracking, site variability, and module degradation with time. An example problem involving the selection of an optimum module glass substrate is presented.

INTRODUCTION

Within the national photovoltaics program sponsored by the U.S. Department of Energy there are a number of activities addressing the optimization of photovoltaic systems and components for a variety of future applications. A significant fraction are involved with design tradeoffs at the system level and associated with determining the true worth of solar energy in comparison with alternative fuels and systems. The most comprehensive of these analyses model the dynamics of solar energy within a utility grid including hourly weather and load modeling. This in-depth level of modeling is providing needed insight into the true characteristics of solar systems and the true value of the generated energy.

A second level of analysis being conducted is focused at photovoltaic system configurations, with the objective of selecting the optimum subsystem characteristics. These analyses often use hour-by-hour system simulation programs to model the dynamic operation of the photovoltaic subsystems such as the array, power conditioning, and storage.

At a level below the system configuration tradeoffs is the class of optimization problems addressed by this paper. This set of problems is associated with subsystem and subassembly optimizations which are often associated with design details such as selection of optimum materials and dimensions.

This class of optimization is often carried out within the constraints of interface requirements to produce the lowest cost of highest performing element possible. Such an approach has the advantage of minimizing design interaction across the interface, but may lead to significant system penalties if the cost and performance interaction across the interface is ignored. The challenge is to develop a simple framework for addressing the optimization of subassembly features which still allows the important system interactions to be included. The development of such a method is the subject of this paper.

A FRAMEWORK FOR OPTIMIZATION

The development of an approach or framework for subassembly optimization requires consideration of three important objectives. These include ease of application to detailed design features, flexibility to adapt to a variety of problem types, and incorporation of important system interface interactions. A key first step in meeting the last objective is the proper choice of the objective function to be minimized.

Objective Function Selection

To properly include important system interactions it is necessary that the objective function to be used reflect the true system design objectives. There are a number of alternate system objective functions in common use today:

- Minimum system life-cycle cost per initial kilowatt of system power output.

*This paper presents the results of one phase of research conducted at the Jet Propulsion Laboratory, California Institute of Technology, for the U.S. Department of Energy, by agreement with the National Aeronautics and Space Administration.

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Minimum system life-cycle cost per life-cycle kilowatt-hr of system energy output.

Minimum system initial cost per initial kilowatt of system power output.

Maximum utility profit based on hourly marginal cost of energy, etc.

The advantages and disadvantages of the alternative objectives depend critically on the details of the particular problem being worked. If none of the tradeoffs in the optimization affect the time-dependent behavior of the system, then minimum system initial cost is an appropriate objective function. However, if time-dependent behaviors such as maintenance, replacement, or performance degradation are important, then an objective function which reflects the importance of event timing must be used.

The time-dependent value of money is generally incorporated by using the life-cycle cost of the total photovoltaic power system. The life-cycle cost for a photovoltaic system is basically the initial cost of the entire plant, including interest during construction, and the "present value" or recurrent costs, such as operation and maintenance, which are distributed throughout the life of the plant. A standardized DOE/EPRI methodology exists with a specific method for calculating system life-cycle cost(1).

An equally important consideration, but one having received little emphasis to date, is the time-dependent worth of the power or energy generated. When considering tradeoffs which involve different performance variations with time such as different degradation rates, one must use an objective function which also reflects the time-dependent worth of the plant output. A companion paper treats this subject in some detail(2).

One candidate function which accommodates the variation of plant output with time is the ratio of the life-cycle plant cost to the life-cycle revenue received from the sale of the energy. Mathematically this function can be represented as

\[ \text{Objective Function} = \frac{\sum_{n=0}^{L} C_n (1+k)^{-n}}{\sum_{n=0}^{L} E_n R_n (1+k)^{-n}} \]  

where

- \( C_n \) = cost outlay in year \( n \) (startup-year $)
- \( E_n \) = energy generated in year \( n \) (kw-hr)
- \( R_n \) = sale price of energy (startup-year $/kw-hr)

\( L \) = number of years plant will be operated.

\( k \) = present value discount rate

Intuitively this concept can be interpreted as minimizing the life-cycle investment per unit of life-cycle revenue.

If one assumes a constant sale price of energy \( R \) then Eq. (1) reduces to

\[ R_L = \frac{\sum_{n=0}^{L} C_n (1+k)^{-n}}{\sum_{n=0}^{L} E_n R_n (1+k)^{-n}} \]  

In this equation \( R_L \) is the energy selling price required to obtain a return on investment consistent with the chosen discount rate, if the plant is operated for \( L \) years. An appropriate objective for a module optimization is to minimize this price.

To further explore Eq. (2), consider the hypothetical plant depicted in Fig. 1. In this figure the plant is arbitrarily represented as having a linearly decreasing output together with a gradually increasing O&M cost as the plant ages. Two questions can be asked: What is the minimum selling price of the energy, and what is the plant lifetime?

The questions are addressed in Fig. 2, where the marginal cost per kilowatt-hour \( C_n/E_n \) is plotted together with the life-cycle energy cost \( R_L \) for an example discount rate of 8%. Note that all dollars are constant dollars based on the year of the plant startup, so that the 8% is in excess of the rate of inflation. The marginal cost per kilowatt-hour is the actual operating expense in year \( n \), in startup-year dollars, per kilowatt-hour produced in year \( n \).

Fig. 1. Annual cost and annual energy output for hypothetical plant versus year of operation

B-2
An important observation from Fig. 2 is that the life-cycle energy cost $R_n$ goes through a mini­
mum, and the minimum occurs at the point where the marginal cost curve crosses the life-cycle cost/ 
kW-hr curve. This point defines the practical end­
of-life of the plant. It can be shown mathemati­
cally that if the same discount rate is used for both the cost and energy (revenue) streams, the
minimum life-cycle cost/kW-hr will always coincide with the crossing of the marginal cost/kW-hr curve. 
In other words, operation to the right of the mini­
mum results in annual operating expenses in excess of the annual revenue associated with the minimum.

An additional observation from Fig. 2 is that this plant could be abandoned after 10 years with only a minor increase in the required selling price of the energy over that associated with the optimum lifetime of around 21 years. The reason is the large present-value discount associated with costs and revenues in later years. The drastic reduction associated with the present-value of future costs is illustrated in Fig. 3 for discount rates of 6, 8 and 10%. These curves also represent the plant depreciation with time associated with the chosen discount rate.

The result of this rapid reduction in the present-value of future dollars is a general insen­
sitivity to events such as output degradation, which occur late in the plants life. This fact lends additional credibility to the use of optimi­
izations which are limited to initial costs for cer­
tain problems. For problems where time-dependent behavior is important, use of a methodology such as the life-cycle cost of energy is recommended.

Reducing the Problem to a Workable Form

The chief difficulty in working with the objec­
tive functions described in the preceding section is that they are written directly in terms of annual costs and energies. On the other hand, most engi­
neering data is in terms of module initial costs and performance variations.

For purposes of developing an optimization strategy consider the problem of selecting between two module design options on the basis of minimizing

![Fig. 2. Life-cycle cost per kilowatt-hour versus plant life $L$ and marginal cost versus year of operation $n$ for a hypothetical plant](image)

![Fig. 3. Present-value of future costs or revenues for discount rates of 6, 8 and 10%](image)
\( e_n \) = fraction of initial annual energy in year \( n \)

\( e_{LC} \) = life-cycle summation of \( e_n \)

\( kL \) = discount rate and plant life

Next, it is desirable to expand the initial annual energy production \( E_0 \) in terms of insolation level \( S \), total module area in the plant \( A_M \), module efficiency \( \eta_M \), balance-of-plant efficiency \( \eta_B \), and peak-insolation-hours per year \( H \). Thus,

\[
R_L = \frac{C_{LC}}{S \eta_M \eta_B \eta_{LC}}
\]

(4)

To obtain \( R_L \) more explicitly in terms of module parameters we next expand the life-cycle cost into its module-dependent and module-independent parts and articulate the costs in terms of module area and array power. Thus

\[
R_L = \frac{C_C + C_{MD} + C_{MLC}}{S \eta_B S h_{LC}} + \frac{C_{BLC}}{S h_{LC}}
\]

(5)

where

- \( R_L \) = total system life-cycle energy cost, $/kW-hr
- \( C_M \) = initial module cost per unit area of module, $/m^2 of module
- \( C_{MD} \) = balance of module-dependent system initial cost per unit area of module, $/m^2 of module
- \( C_{MLC} \) = module-dependent life cycle cost exclusive of initial costs, per unit area of module, $/m^2 of module
- \( C_{BLC} \) = total module-independent balance-of-plant life-cycle cost per kilowatt of total plant output power at insolation \( S \) and NOCT, $/P_kkW of plant output
- \( \eta_M \) = module efficiency (power output per unit of total module area at insolation \( S \) and NOCT, divided by \( S \))
- \( \eta_B \) = balance-of-plant efficiency (average plant power output divided by array power input)
- \( S \) = reference insolation level, kW/m^2
- \( H \) = peak-insolation-hours per year captured by the array (Langleyes/day divided by \( S \), mW/cm^2, times 423.4), hr/yr
- \( e_{LC} \) = life-cycle summation of annual fraction of initial energy output

\( NOCT \) = nominal operating cell temperature with module fluid instantiated, °C

Equation (5) is a particularly useful form for many optimization problems.

As an additional aide for problems associated with internal module parameters it is also useful to expand the module cost \( C_M \) into three component parts: cell-related costs, encapsulant related costs, and fixed costs. Thus

\[
C_M = C_C n_F + C_E + C_F
\]

(6)

where

- \( C_C \) = solar-cell-related cost per module, $/m^2 of cell
- \( C_E \) = encapsulant-related cost per module, $/m^2 of module
- \( C_F \) = fixed cost per module ($/module)
- \( n_F \) = module packing efficiency (total cell area per unit area of module divided by total module area)
- \( A \) = total area of module, m^2

Solving the Example

To illustrate the method suggested by Eqs. (5) and (6), consider its application to the example problem of the two types of glass. The critical first step is to articulate the parameter dependencies; i.e. which of the parameters in Eqs. (5) and (6) are dependent on the choice of glass. This step can be greatly simplified by properly posing the problem.

In the example the two glass types are considered to have different strengths per thickness (tempered and untempered) and different transmissiom losses per unit thickness. The required glass thickness is therefore a critical parameter. To obtain comparable results, either a uniform design criteria must be applied or the lack of uniformity must be explicitly dealt with.

To simplify the problem, consider the design criteria to be that both modules will have equal resistance to damage and degradation so that maintenance costs are held constant. The thickness of the glass is therefore determined by the glass strength, the structural loading design level, and the module size.

Unfortunately an unwanted degree of freedom still exists at this point; i.e., the glass thickness is dependent on the module size assumed. Several candidate strategies for eliminating this degree of freedom include:
• hold the glass thickness constant
• hold the module power constant
• hold the module size constant

Each of these constraints will lead to different dependencies between the parameters.

Holding the module size constant is chosen because it minimizes these dependencies. Changing the size would have altered the module frame, the installation cost, the module packing efficiency and many dependencies difficult to estimate. However, with the chosen constraints — constant size and environmental durability — it is possible that the cheapest tempered glass available exceeds the durability design criteria. This may be acknowledged by reducing the maintenance cost an appropriate amount.

Increasing the module size to fully utilize the stronger glass must be approached with extreme caution because of the difficulty in estimating the effects of size on manufacturing cost, shipping cost, handling cost, etc. The natural tendency is to assume these costs are insensitive to size. This results in the option with fewer, larger modules being nearly always cheaper. Watch out!

The remaining dependencies in the example relate to optical transmission losses associated with the glass thickness, the unequal soiling assumed, and the cost difference between the glasses. Note that although the optical transmission difference is included in the module efficiency $n_m$, the optical loss due to soiling is, by convention, a system loss, and included in the balance-of-plant efficiency $n_B$. As an alternative the dust loss could be included in the life-cycle energy fraction $\epsilon_{LC}$.

Table 1 summarizes the set of hypothetical dependencies for the example problem and summarizes the resulting module cost and life-cycle energy cost calculated using Eqs. (5) and (6). A life-cycle energy fraction of 10 is assumed on the basis of Fig. 4. Notice that this value is fairly insensitive to life beyond 20 years.

From the bottom line in Table 1 it can be seen that the optimum choice from a module-cost point-of-view is not the proper choice from the standpoint of lowest system energy cost.

AN ALTERNATE COST-BENEFIT APPROACH

A disadvantage of the approach used in the preceding example is that a large number of poorly known parameters exist, such as $\epsilon_{LC}$, $n$, and $\epsilon_{MC}$, which are likely to be independent of the question at hand.

An alternate strategy is to calculate the sensitivity of $R_L$ to changes in the dependent parameters, while holding the independent parameters fixed. A beneficial design trade is then defined as one where the incremental benefit is larger than the incremental expense; i.e. the incremental change in $R_L$ is negative. This can be expressed mathematically as

$$\frac{\partial R_L}{\partial C} < 0$$  \hspace{1cm} (7)$$

The critical step in this approach is to correctly take the partial derivative of Eq. (5) with respect to the principal design parameter so that all interdependencies are properly accounted for.

To illustrate this approach, again consider the example problem with the two types of glass. As the first step we choose the glass cost $C$ as the principal design parameter denoting the glass type. With this selection the problem reduces to taking the derivative of Eq. (5) with respect to $C$ while holding all independent parameters fixed. The independent parameters are noted by an absence of a bullet in the right hand column of Table 1. Letting $S = 1$ we get

$$\frac{\partial R_L}{\partial C} \left( n_A n_B \frac{H_{LC}}{H} \right) = \frac{3 n_A n_B}{H} \left( \frac{n_A n_B}{H_{LC}} \right) \left( n_M n_B \frac{H_{LC}}{H} \right) \left( n_{MC} n_B \frac{H}{H_{LC}} \right) \left( n_M n_B \frac{H}{H_{LC}} \right) = 0$$

Setting this expression to zero gives:

$$\frac{3 n_A n_B}{H} = \left( n_M n_B \frac{H_{LC}}{H} \right) \left( n_M n_B \frac{H_{LC}}{H} \right) \left( n_M n_B \frac{H_{LC}}{H} \right) \left( n_M n_B \frac{H_{LC}}{H} \right)$$

Fig. 4. Life-cycle energy fraction assuming no degradation with time (present-value of an annuity of 1) versus life.
Table 1. Example parameter dependency for two types of glass superstrates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Annealed</th>
<th>Tempered</th>
<th>Units</th>
<th>Dependent parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass thickness</td>
<td>C</td>
<td>3.0</td>
<td>2.0</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>Optical transmission</td>
<td>nB</td>
<td>0.801</td>
<td>0.837</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dust transmission</td>
<td>nEC</td>
<td>0.133</td>
<td>0.138</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass cost</td>
<td>nF</td>
<td>0.90</td>
<td>0.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balance-of-plant efficiency</td>
<td>nM</td>
<td>0.120</td>
<td>0.119</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Encapsulated cell efficiency</td>
<td>60</td>
<td>60</td>
<td></td>
<td>$/m^2</td>
<td></td>
</tr>
<tr>
<td>Packing efficiency</td>
<td>4.00</td>
<td>7.00</td>
<td></td>
<td>$/m^2</td>
<td></td>
</tr>
<tr>
<td>Module efficiency</td>
<td>4.80</td>
<td>4.80</td>
<td></td>
<td>$</td>
<td></td>
</tr>
<tr>
<td>Cell-related cost</td>
<td>A</td>
<td>3.2</td>
<td>3.2</td>
<td>m^2</td>
<td></td>
</tr>
<tr>
<td>Encapsulant cost</td>
<td>CMD</td>
<td>30.0</td>
<td>30.0</td>
<td>$/m^2</td>
<td></td>
</tr>
<tr>
<td>Module-dependent O&amp;M cost</td>
<td>CMCL</td>
<td>10.0</td>
<td>10.0</td>
<td>$/m^2</td>
<td></td>
</tr>
<tr>
<td>Other system life-cycle</td>
<td>CMLC</td>
<td>100</td>
<td>100</td>
<td>$/kW</td>
<td></td>
</tr>
<tr>
<td>Module cost per m^2</td>
<td>CM</td>
<td>59.5</td>
<td>62.5</td>
<td>$/m^2</td>
<td></td>
</tr>
<tr>
<td>Peak hours per year</td>
<td>H</td>
<td>1825</td>
<td>1825</td>
<td>hr</td>
<td></td>
</tr>
<tr>
<td>Life-cycle energy fraction</td>
<td>eLC</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Insolation level</td>
<td>S</td>
<td>1</td>
<td>1</td>
<td>kW/m^2</td>
<td></td>
</tr>
<tr>
<td>Discount rate</td>
<td>k</td>
<td>0.08</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plant life</td>
<td>L</td>
<td>30</td>
<td>30</td>
<td>yr</td>
<td></td>
</tr>
<tr>
<td>Module cost</td>
<td>C_t/nM</td>
<td>496</td>
<td>525</td>
<td>$/kW</td>
<td></td>
</tr>
<tr>
<td>Life-cycle energy cost</td>
<td>R_L</td>
<td>0.0622</td>
<td>0.0619</td>
<td>$/kW-hr</td>
<td></td>
</tr>
</tbody>
</table>

Solving for ∂C_M/∂C from Eq. (6) gives:

\[
\frac{\partial C_M}{\partial C} = \frac{\partial C_E}{\partial C} = 1
\]

(9)

If we combine Eqs. (8) and (9) and consider the differential changes as deltas we obtain:

\[
\Delta C = \left( C_M + C_{HD} + C_{MLC} \right) \left( \frac{\Delta n_B}{n_B} + \frac{\Delta n_M}{n_M} \right)
\]

(10)

Equation (10) states that for the example problem the energy cost will decrease if the delta glass cost is less than the right-hand-side expression.

Substituting the values from Table 1 indicates that the tempered glass will be best if

\[
\Delta C \leq \left( 59.5 + 30 + 10 \right) \left( \frac{0.036}{0.801} - \frac{0.001}{0.120} \right) = 3.64 \$/m^2
\]

Since the increased cost of the tempered glass is only 3 $/m^2, the tempered glass is best.

**Additional Design Parameters**

Using the above approach, additional cost-benefit relationships can be easily derived for use as design tools when the need arises. As an aid, some of the more commonly encountered problems have...
been worked and the results are presented in Table 2. All of these problems assume the system parameters $n_B$, $n_H$, $n_{LC}$ and $n_{MLC}$ to be independent of the principal design parameter. For a detailed description of the efficiency terminology used in Table 1 the reader is referred to Ref. (3).

**CONCLUSIONS**

A review of system optimization objective functions indicates that minimum system life-cycle cost per life-cycle energy output is a useful function for subassembly optimization, particularly when time-dependent parameters are involved. An advantage of this function is its ability to reflect the system performance sensitivity to energy-related effects such as those associated with site variability, solar tracking, and performance degradation over time. An important design tool for module optimization has been obtained by reducing this function to a form which allows easy application to the detailed design features typically encountered with photovoltaic modules.

**REFERENCES**


<table>
<thead>
<tr>
<th>Design parameter</th>
<th>Benefit criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell efficiency</td>
<td>$\Delta C = \frac{\Delta n_C}{\eta_C} \left( C_M + C_{MD} + C_{MLC} \right)$</td>
</tr>
<tr>
<td>Cell mismatch</td>
<td>$\Delta C = \frac{\Delta n_{MIS}}{\eta_{MIS}} \left( C_M + C_{MD} + C_{MLC} \right)$</td>
</tr>
<tr>
<td>Optical transmission</td>
<td>$\Delta C = \frac{\Delta n_T}{\eta_T} \left( C_M + C_{MD} + C_{MLC} \right)$</td>
</tr>
<tr>
<td>Operating temperature</td>
<td>$\Delta C = \frac{\Delta n_{NOCT}}{\eta_{NOCT}} \left( C_M + C_{MD} + C_{MLC} \right)$</td>
</tr>
<tr>
<td>Cell shape</td>
<td>$\Delta C = \frac{\Delta n_H}{\eta_H} \left( \frac{C_P}{n_A} + C_E + C_{MD} + C_{MLC} \right)$</td>
</tr>
<tr>
<td>Border/buss area</td>
<td>$\Delta C = \frac{\Delta n_{BR}}{\eta_{BR}} \left( C_P + C_{CA} + C_{MD} + C_{MLC} \right)$</td>
</tr>
</tbody>
</table>

where:

- $\eta_M$ = overall module efficiency at 100 mW/cm$^2$, NOCT
- $\eta_P$ = $\eta_{NOCT}$ x $\eta_{EC}$
- $\eta_{MIS}$ = module packing efficiency = $\eta_{BR}$ x $\eta_N$
- $\eta_{BR}$ = module border/buss/interconnect area efficiency
- $\eta_N$ = cell nesting efficiency
- $\eta_{NOCT}$ = encapsulated cell efficiency at 100 mW/cm$^2$, 28°C
- $\eta_{EC}$ = $\eta_C$ x $\eta_T$ x $\eta_{MIS}$
- $\eta_C$ = bare cell efficiency (100 mW/cm$^2$, 28°C)
- $\eta_T$ = optical transmission efficiency
- $\eta_{MIS}$ = electrical mismatch/series resistance efficiency

*For other definitions see Eqs. (5) and (6) in text.*
FLAT-PLATE PHOTOVOLTAIC ARRAY DESIGN OPTIMIZATION

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SUMMARY
Specific analyses have been conducted in the areas of structural design optimization, optimization of array series/parallel circuit design, thermal design optimization, optimization of environmental protection features, and others. This paper integrates the results from these various studies and draws general conclusions relative to optimal features for future modules. The described analysis is based on minimizing the total photovoltaic system life-cycle energy cost including repair and replacement of failed cells and modules. The conclusions presented provide useful design guidelines for designers of future flat-plate photovoltaic modules.

INTRODUCTION
A comprehensive program of flat-plate solar array design optimization is being carried out as part of the Jet Propulsion Laboratory's Low-cost Solar Array Project. The objective of these studies is to define means of reducing the cost and improving the utility and reliability of photovoltaic flat-plate arrays for the broad spectrum of terrestrial applications. Specific analyses have been conducted in the areas of structural design optimization, optimization of array series/parallel circuit design, thermal design optimization, optimization of environmental protection features, and others.

This paper describes a key analysis which serves to integrate many of these ongoing studies and is based on minimizing the total PV system life-cycle energy cost including repair and replacement of failed cells and modules. This analysis directly integrates array structures costs, panel costs, module costs, replacement strategies, series/paralleling tradeoffs, module size tradeoffs, cell reliability performance, and several other factors.

THE PROBLEM
The primary objective of the described analysis is to provide a means of integrating the results from a variety of diverse flat-plate solar array design studies and to draw bottom-line conclusions relative to optimum module and array mechanical and electrical circuit configurations. Because of the strong interaction between module size and replacement cost, any analysis of module size is forced to also consider the expected degree and timing of module replacement. This is further tied to the entire question of module reliability, definition of module failure and replacement criteria, and the reliability engineering features, such as series/parallel, by-pass diodes, and redundant solar cell electrical contacts. Looked at the other way around, any analysis for the selection of the optimal reliability engineering features must also consider the costs of replacement, including the size of the modular replacement unit.

To illustrate the nature of the problem consider the selection of the optimum mechanical and electrical configuration for a flat-plate module for a large ground-mounted photovoltaic array. A complete analysis should, as a minimum, address the following interactions:

- Module superstrate (glass) thickness and material cost versus size
- Module efficiency (perimeter area effect and encapsulant transmission) versus size
- Module efficiency loss due to cell mismatch versus circuit configuration
- Module assembly cost versus size and circuit configuration
- Module manufacturing yield cost (larger modules have higher probability of containing faulty parts, but greater circuit redundancy reduces losses associated with faults)
- Module shipping and handling and installation cost versus size
- Support structure cost versus module size and efficiency

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* This paper presents the results of one phase of research conducted at the Jet Propulsion Laboratory, California Institute of Technology, for the U.S. Department of Energy, by agreement with the National Aeronautics and Space Administration.
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Presented at the 14th IEEE Photovoltaic Specialists Conference, San Diego, California, January 7-10, 1980

C-1
The optimization is formulated by setting the strategy.

where:

- System power degradation versus electrical circuit reliability as influenced by series/paralleling, by-pass diodes, multiple cell contacts, etc.
- Module life-cycle replacement cost versus module size and system power degradation

Review of the above interactions indicates the complexity of the problem at hand.

APPROACH

The approach utilized to solve the above problem first involves generating parametric data defining the cost/performance dependency associated with each of the bulleted interactions except the last. This work is basically complete and has resulted from a variety of JPL inhouse configuration and series/paralleling studies and power-system/support-structure/maintenance-cost studies conducted by Bechtel Corporation under contract to JPL (1), (2), (3), (4), (5). Details of these results will be examined later in this paper.

The next step in the solution is the construction of an overall cost optimization algorithm based on minimizing the total system life-cycle energy cost including selection of the optimum module replacement strategy. This has been accomplished using a multi-variable optimization program which first computes the system life-cycle energy costs using different module replacement strategies until the least-cost strategy is identified. The analysis is then repeated for each alternative system configuration (module size and electrical circuit) to allow selection of the least-cost total system design including the module replacement strategy.

The optimization is formulated by setting the life-cycle benefits equal to the life-cycle costs including module replacement over the life of the plant. In mathematical form the derivation follows this author's previous work (6):

\[
\text{Life-cycle benefit} = \sum_{i=0}^{20} E_i (1 + k)^{-i}
\]

\[
\text{Life-cycle cost} = C_0 + \sum_{i=1}^{20} C_i M_i (1 + k)^{-i}
\]

where:

- \( R \) = Cost (worth) of energy assumed constant over the plant lifetime (startup-year $/KW-hr)
- \( E_i \) = Energy generated in year \( i \) (KW-hrs)
- \( C_0 \) = Initial plant cost (startup-year $)
- \( C_i \) = Cost per module replacement action (startup-year $/module)
- \( M_i \) = Number of modules replaced in year \( i \)
- \( k \) = Present value discount rate
- \( 20 \) = Plant lifetime (years)

The optimum system design is then found by minimizing the breakeven cost of the photovoltaic energy which is given by:

\[
R = \frac{C_0 + \sum_{i=1}^{20} C_i M_i (1 + k)^{-i}}{1 + \sum_{i=0}^{20} E_i (1 + k)^{-1}}
\]

Two numerical algorithms have been successfully used to perform the actual minimization at JPL. The first uses a multivariable optimization program written by this author based on the simplex method of Nelder and Mead (7). This program repeatedly evaluates any arbitrary function of \( n \) variables and locates the values of the variables where the function is minimum. For the problem at hand the function to be minimized is equation (1) and the \( n \) variables are the 20 values of \( M_i \) representing the number of modules replaced in each of the twenty years of the photovoltaic system's life.

The simplex algorithm has the advantage of being able to locate the least-cost replacement strategy independent of its complexity. However, it suffers the disadvantage of converging very slowly. An important finding from the use of this algorithm is that in nearly all cases analyzed the optimum replacement strategy has been either no module replacement at all, or module replacement each time a solar cell fails. In the rare cases where one of these two options has not been optimum, the optimum replacement strategy has always been to fully replace failed modules in the first few years of the system's life, and then to replace no modules in subsequent years.

Based on this finding a second optimization algorithm has been developed based on selecting the least cost of 21 trial replacement strategies. The 21 trial strategies include no replacement at all, and module replacement for each cell that fails during the first through the \( r \)th year \( (r = 1, 20) \) with no replacement for the balance of the plant's 20-year life. This algorithm works very efficiently and is the one currently being used at JPL.

The remainder of this paper is devoted to implementing this algorithm for the case of large multi-megawatt ground-mounted arrays similar to those that might be used in a large industrial or central station application. Although the number of design alternatives considered has been limited for presentation purposes, the approach is being applied to a much more exhaustive set of alternatives including residential applications as part of the ongoing JPL/LSA Engineering activities.

COST DEPENDENCIES FOR LARGE GROUND-MOUNTED ARRAYS

As a first step in the analysis it is necessary to define the cost/performance dependency associated with each of the important interactions bulleted earlier in this paper. Bechtel Corporation, in their work for JPL, has developed a number of
Fig. 1. Definition of array elements

In their work Bechtel has examined three module sizes (2 x 4 foot, 4 x 4 foot, and 4 x 8 foot) and a number of alternative support-structure configurations. For lowest installed cost their analysis indicates that modules should be factory-mounted and shipped in structural panels which become integral parts of the array structure as shown in Figure 1. Although they found the areal cost of the glass to be the same for each of the three module sizes (because of minimum gauge considerations), the cost of the panel structure was found to be strongly dependent on the module size. On the other hand, the total structural cost was found to vary little between alternative panel sizes and support-structure configurations for a given module size.

Based on this last finding a single frame/support structure configuration and single panel size (8 x 16 foot) is used throughout the remainder of this paper.

Bechtel also developed a variety of installation and replacement cost scenarios. Their least cost scenario, which is adopted in this paper, involves locating the faulty panel in the field and exchanging the panel with a new or rebuilt one. The faulty panel is then returned to a repair station where the faulty module is located within the panel and is replaced with a new module. The rebuilt panel is now ready for reuse in the field. Table I summarizes these cost dependencies.

Also included in Table I is the cost for the photovoltaic module (a glass solar cell sandwich with no frame) including a term referred to as module yield cost. The module yield cost is the amount that must be added to the price of a module to pay for modules scrapped during module final assembly, shipping and installation due to broken cells and other circuit failures. The module failure criteria is based on controlling electrical mismatch in the array and assumes that a module is rejected if its power loss is greater than 10 percent of the average peak power output for all modules. The yield cost value in Table I is for a circuit failure density of 1 per 1000 solar cells, and is dependent on the level of module circuit redundancy. Figure 2 summarizes the yield figures computed for this failure density as a function of module series/paralleling for three sizes of modules (2). Because automated soldering techniques are assumed, no additional module cost is associated with the increased internal series/paralleling.

Another important area of cost dependency involves parameters which alter module or array electrical efficiency. Changing efficiency directly leverages the total quantity of modules and support structure required, and thus directly impacts the initial plant cost. Two efficiency dependencies are important in the present analysis: a) decreased module packing efficiency due to increased border on smaller modules (1), and b) decreased cell mismatch losses associated with high degrees of series/paralleling (3). The effect of these and other system performance dependencies are summarized in Table II.

Except for the module efficiency entries, the majority of the figures in Table II reflect nominal values required in the life-cycle energy cost calculations and have little impact on the relative comparisons which result from the analyses.

ARRAY DEGRADATION VERSUS CIRCUIT REDUNDANCY

The remaining dependency which must be examined is the effect of electrical circuit redundancy on array power degradation resulting from field failures. Before discussing the specifics of this subject it is useful to first introduce some circuit redundancy concepts and nomenclature.

Table 1. Cost Dependencies for Array Elements

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>UNITS</th>
<th>MODULE SIZE</th>
<th>2 x 4</th>
<th>4 x 4</th>
<th>4 x 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>INITIAL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MODULE DIRECT COST</td>
<td>$/m²</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>MODULE YIELD COST*</td>
<td>$/m²</td>
<td>0-5</td>
<td>0-8</td>
<td>0-23</td>
<td></td>
</tr>
<tr>
<td>* MODULE SUBTOTAL</td>
<td>$/m²</td>
<td>60-95</td>
<td>60-68</td>
<td>60-83</td>
<td></td>
</tr>
<tr>
<td>PANEL FRAMES</td>
<td>$/m²</td>
<td>18</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PANEL WIRING</td>
<td>$/m²</td>
<td>2-4</td>
<td>2-3</td>
<td>1-2</td>
<td></td>
</tr>
<tr>
<td>PANEL SUBTOTAL</td>
<td>$/m²</td>
<td>26-28</td>
<td>20-21</td>
<td>16-17</td>
<td></td>
</tr>
<tr>
<td>PANEL INSTALLATION</td>
<td>$/m²</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INSTALLED ARRAY STRUCT</td>
<td>$/m²</td>
<td>22</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>* ARRAY TOTAL</td>
<td>$/m²</td>
<td>109-116</td>
<td>103-112</td>
<td>99-123</td>
<td></td>
</tr>
<tr>
<td>REPLACEMENT ACTION</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FAULT IDENTIFICATION</td>
<td>$/CELL</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>PANEL SUBSTITUTION LABOR</td>
<td>$/PAN</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>MODULE REPLACEMENT LABOR</td>
<td>$/MODULE</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>REPLACEMENT MODULE PARTS (INC 1% INVENTORY COST)</td>
<td>$/m²</td>
<td>61-89</td>
<td>61-69</td>
<td>61-84</td>
<td></td>
</tr>
</tbody>
</table>

*1 CELL FAILURE PER 1000 DURING ASSEMBLY/SHPING/INSTALLATION

Fig. 2. Module yield versus series paralleling assuming one failure per 1000 cells and module rejection for power loss greater than 10 percent.
From a variety of overall photovoltaic system studies including work by Bechtel and others there is good agreement that large centralized power systems will be logically subdivided into a number of individual array subfields in the 2 to 10 megawatt size range (4). It has also been determined that the optimum DC voltage level for a subfield of this size will be somewhere in the 1000 to 2000 volt range so as to control $i^2R$ losses in the field wiring and power conditioner (4). Because each solar cell produces an output of approximately 1.0 watt at 0.5 volts, it can be seen that each power conditioner will be fed by an approximately square matrix of from 2 to 10 million individual solar cells.

The first step toward circuit redundancy is generally associated with dividing this large matrix of cells into a number of parallel solar cell networks referred to as "branch circuits." The branch circuits provide convenient points for monitoring array performance and provide an ability to isolate small areas of the total array for maintenance and repair. As shown in Figure 3 each branch circuit may contain a single string of serial solar cells or many strings of serial solar cells interconnected periodically by cross ties. The cross ties divide each branch circuit into a number of "series blocks."

It is the series/paralleling of the individual branch circuits which is key to controlling array degradation. Three parameters are of particular importance—the number of parallel strings, the number of series blocks, and the number of cells per substring within each series block. For any specific branch circuit configuration the substring failure density ($F_{ss}$) can be easily computed from the cell failure density ($F_c$) and the number of cells per substring ($n$) using the following statistical equation:

$$F_{ss} = 1 - (1 - F_c)^n \quad (2)$$

During operation of the life-cycle optimization program the cell failure density present in the array at any point in its life is computed by summing the net failures to date due to the assumed failure rate together with the effect of module replacements in preceding years. For the analyses which follow the cell failure rate has been set equal to a constant value of one open-circuit cell failure per 10,000 cells per year. This value is felt to be a reasonable expectation for future large scale arrays and is only a factor of two or so better than is currently being experienced in the 25 kW Mead, Nebraska, experiment. A failure rate of 1 per 1000 per year is also examined to indicate the sensitivity of selected results to the failure rate.

A major computational difficulty is the task of computing the array power degradation for each substring failure density level and for the variety of array series/parallel/diode electrical circuit configurations of interest. A major activity at JPL has been addressed to this problem over the last year and has led to the development of an elaborate parametric analysis described in detail in a companion paper (2). The results from this analysis are entered into the life-cycle cost optimization program and are used to compute the system power output each year of its 20-year life based on the net cell failures per year. An example plot defining the dependence between average branch circuit power loss and substring failure density for series/parallel circuits involving 8 parallel strings and various numbers of series blocks is shown in Figure 4.

Table 2. Nominal Life-Cycle Performance Parameters

<table>
<thead>
<tr>
<th>INITIAL ARRAY EFFICIENCY</th>
<th>MODULE SIZE (n x n)</th>
<th>2 x 4</th>
<th>4 x 4</th>
<th>4 x 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENCAP. CELL EFFICIENCY</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>NOCT EFFICIENCY</td>
<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>PACKING EFFICIENCY</td>
<td>0.89</td>
<td>0.91</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>ARRAY EFFICIENCY SUBTOTAL</td>
<td>0.123</td>
<td>0.126</td>
<td>0.128</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BALANCE-OF-PLANT EFFICIENCY</th>
<th>ELECTRICAL EFFICIENCY</th>
<th>0.92</th>
</tr>
</thead>
<tbody>
<tr>
<td>BALANCE-OF-PLANT SUBTOTAL</td>
<td>MODULE SOILING EFFICIENCY</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>BALANCE-OF-PLANT SUBTOTAL</td>
<td>0.85</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BALANCE-OF-PLANT COSTS (1975$)</th>
<th>150 $kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>DISCOUNT RATE (OVER INFLATION)</td>
<td>10%</td>
</tr>
<tr>
<td>ANNUAL INSOLATION</td>
<td>1825 kW-h/m²-yr</td>
</tr>
</tbody>
</table>

Fig. 3. Series/parallel nomenclature

To set the stage for the life-cycle analyses which follow in the next section, it is instructive to consider the problem of calculating the expected power degradation for a 1000 volt, large industrial array made up of 8 parallel-string branch circuits as considered in Figure 4. To achieve the 1000 volt nominal operating voltage requires approximately 2400 series solar cells, and thus a branch circuit of 2400 series by eight parallel. If 14 cross ties are incorporated into each branch circuit, there will be 15 series blocks per branch circuit and 2400/15 = 160 cells per substring.

To utilize Figure 4 it is first necessary to compute the expected substring failure density for the time of interest. As an example, the sub-

C-4
Fig. 4. Power loss relationships for 8-parallel-string branch circuits

Substring failure density at the end of five years with no module replacements and a constant cell failure rate of 0.0001 per year is found using equation (2) as:

$$F_{as} = 1 - (1 - 0.0005)^{160} = 0.0769$$

Using this value and interpolating in Figure 4 gives an expected array degradation of approximately 16 percent after 5 years. Figure 5 expands on this result to illustrate the expected degradation in subsequent years and the result of different numbers of series blocks per branch circuit.

**LIFE-CYCLE ANALYSIS RESULTS**

As an example application, the life-cycle optimization algorithm is now used to calculate the life-cycle cost tradeoffs associated with the previous example which incorporated branch circuits with 8 parallel by 2400 series cells and a uniform cell failure rate of 0.0001 per year. In addition, the analysis is initially focused on the use of 4 by 8 foot modules, each containing 320 solar cells, and with the cost dependencies previously developed in Tables I and II. For these assumptions Figure 6 displays the calculated life-cycle energy costs as a function of the number of series blocks per branch circuit and for two replacement strategies. In the first strategy no module replacement is allowed and it can be seen that the life-cycle costs increase sharply with low numbers of series blocks. This reflects the rapid array degradation exhibited in Figure 5 for these circuit configurations.

For the second curve in Figure 6 modules are replaced each time a solar cell fails during the 20-year life of the plant. This results in no power degradation, but does cause a substantial module replacement-cost contribution. This cost also varies with the number of series blocks due to reductions in module yield costs which occur when module series/paralleling achieves 8 parallel by two or more series blocks. This degree of module series/paralleling is only possible when 120 or more series blocks are used per branch circuit.

As seen in Figure 6 the optimum maintenance strategy depends on the degree of series/parallel- ling. When low degrees of series/paralleling are used, the least-cost maintenance strategy is to replace the affected module each time a solar cell fails. On the other hand, when a high degree of series/paralleling is used, the least-cost strategy involves no module replacement. Only in a very small region where the two curves cross is a partial-replacement strategy optimum.

In future graphs only the optimum-maintenance (least life-cycle) cost is plotted for each number of series blocks per branch circuit.

When considering Figure 6 it is apparent that the optimum configuration for an array of 4 by 8 foot modules in 8-parallel-string branch circuits is 240 or more series blocks, with no module replace-
Fig. 7. Minimum life-cycle energy costs for various module sizes

Figure 7 compares this result with similar results for 2 by 4 and 4 by 4 foot modules. As seen in Figure 7 both smaller module sizes result in higher system energy costs because of the higher support structure cost for small modules. Also, the cost reduction due to yield-cost improvements occurs at a higher number of series blocks because of the fewer cells per module.

If, for some reason, a low degree of series/paralleling is utilized together with a full replacement strategy, the smaller modules are preferred over the larger 4 by 8 foot modules. This is because of the higher per unit replacement cost for large modules and the similarly higher yield costs when no module internal series/paralleling can be utilized.

Figure 8 illustrates the key argument against the adoption of this low-series/paralleling, full-replacement strategy by indicating the effect of a higher cell failure rate. As can be seen the low-series/paralleling configurations are much more sensitive to higher than expected failure rates than are the high-series/paralleling configurations.

Fig. 8. Minimum life-cycle energy cost versus failure rate

Fig. 9. Minimum life-cycle energy cost versus cell fill factor

Figure 9 illustrates another factor which can lead to faster than normal array degradation—cell or module fill factor. Higher fill factors cause the operating (maximum-power) current level to be closer to the array short-circuit current level. As a result, a reduction in current carrying capability due to a cell failure is more likely to lead to current limiting and reverse biasing with high-fill-factor cells and modules. The result is faster array degradation, and higher costs as shown in Figure 9.

Figure 10 expands the parametric study to include the effects of other choices for the number of parallel strings—in this case 4 and 16 strings in parallel. The number of parallel strings is found to have little influence on the overall conclusions relative to the optimum number of series blocks, the optimum module size, or the optimum maintenance strategy. As can be seen the primary effect is somewhat lower array degradation with correspondingly lower life-cycle energy costs for higher degrees of paralleling in the 100 series-block region of the plot. The higher degree of paralleling also has the advantage of minimizing hot-spot heating due to reverse-biasing effects.

As indicated in an earlier paper by this author, hot-spot heating increases with increasing numbers of series blocks (1). For present day solar cells with high shunt resistances the upper limit on the allowable number of series blocks is approximately 10 to 15. This effect rules out the use of the
large number of series blocks suggested by the earlier figures unless a corrective action such as by-pass diodes is incorporated into each branch circuit. This is indeed the recommended approach.

Figure 11 describes the life-cycle costs for a variety of branch circuit configurations incorporating a by-pass diode around each series block. Note that a single series string with diodes provides the least life-cycle cost followed closely by branch circuits with 8 or more parallel strings with diodes. To protect against hot-spot heating due to partial cell loss or shadowing, it is also desirable to limit the number of series cells per diode to 15 or less. For the example at hand with one diode per series block this further restriction requires that the number of series blocks be 160 or greater. The least life-cycle cost configuration which also maintains acceptable hot-spot heating levels is thus one with at least 160 series blocks and 160 by-pass diodes per branch circuit. A moderate degree of paralleling (8 or more) is useful to limit the total number of diodes required, and to achieve a reasonable number of branch circuits per power conditioner.

CONCLUSIONS

The approach of minimizing life-cycle cost over life-cycle energy has been shown to be a useful technique for array optimization, particularly when time-dependent parameters such as array degradation and maintenance are involved. The technique provides the necessary algorithm for integrating diverse attribute dependencies and drawing bottom-line conclusions relative to array configuration tradeoffs. Data have been presented which show that the life-cycle cost for large ground-mounted arrays can be significantly reduced by selecting the optimum mechanical and electrical circuit configurations. Key factors include the incorporation of large modules to reduce support structure cost, and the incorporation of extensive series/paralleling and diodes to reduce module yield costs and eliminate the need for module field replacement.

REFERENCES


POWER LOSS IN PHOTOVOLTAIC ARRAYS DUE TO MISMATCH IN CELL CHARACTERISTICS†

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Abstract—Variations in the current-voltage characteristics of photovoltaic cells can lead to significant power loss “due to mismatch” when the cells are connected together in a network. This study explores how this mismatch loss depends on variations in max-power current and max-power voltage from cell to cell. An analysis of a series string is first performed. Losses in a parallel string are also determined. Estimates of mismatch losses in more complex arrays are then obtained. In addition to generally excellent comparison with several numerical studies, results show that, for a series string, there exists a critical magnitude of deviation in cell max-power current beyond which the power loss due to mismatch is sensitive to both the number of cells placed in series and the shape of the probability density function defining variations in max-power current. This critical level also depends on the cell fill-factor.

I. INTRODUCTION

Variations in the current-voltage characteristics of photovoltaic cells can lead to a power loss “due to mismatch” when the cells are connected in series and parallel within a network. That is, the maximum output power available from the network, will be less than the sum of the maximum output powers of the individual cells, given that all operate at common cell-temperature and light-intensity levels. The objective of this study is to determine how this mismatch loss depends on variations in cell characteristics. In particular, given statistical distributions of the current and the voltage of the individual cells operating at maximum power, again at some standard reference conditions for cell temperature and light intensity, we seek to derive an expression for the expected value of the maximum power available from the array. This, in turn, will help answer another more practical question: how much variation in cell characteristics can be tolerated before individual cells must be sorted in order to meet module or array performance goals?

We proceed as follows: We model the behavior of the cell in the vicinity of the max-power point by a relatively simple expression for the cell current as a function of cell voltage. Two parameters, defined by the max-power current and max-power voltage appear in this relationship. We allow for small variations, say of order ε, in cell max-power current and voltage with respect to their mean values, derive expressions for the network max-power current and voltage. Then, by means of an expansion on ε, obtain explicit expressions for these network characteristics in terms of the given small variations in cell characteristics. Results are first obtained for a series string of cells, then for a parallel string. Estimates of mismatch losses for more complex networks are obtained from a synthesis of results for these two basic networks. The analysis is carried through for two different ranges of variations in cell max-power current and max-power voltage.

†This work was sponsored by the U.S. Department of Energy.
The last alternative was followed here. The fill factor is defined as:

$$f_f = \frac{(I_mp V_mp)}{(I_sc V_oc)}$$

(6)

where \(I_{sc}\) is the cell's short-circuit current and \(V_{oc}\) is its open-circuit current. For a nominal cell we set \(V_{mp} = V_{oc}\) and \(I_{mp} = I_{sc}\), and obtain from (1):

$$\frac{I_{sc}}{I_{mp}} = \alpha - \beta = \alpha = \frac{(1 + c)}{c}$$

(7)

$$\frac{V_{oc}}{V_{mp}} = \frac{[c + \ln (1 + c)]}{c}$$

(8)

In this process we have neglected \(e^{-c}\) with respect to \(I_{sc}\), a step justified by our knowledge of realistic values for \(c\).

The fill factor, in terms of \(c\), is then:

$$f_f = \frac{c^2}{(1 + c)[c + \ln (1 + c)]}$$

(9)

Figure 1 shows \(c\) as a function of \(f_f\). As the fill-factor increases, \(c\) also increases, the maximum power available from the cell approaches the product of short-circuit current and open-circuit voltage. Given \(f_f\), (9) fixes \(c\), and (7) and (8) in turn, yield values for the ratios \(I_{sc}/I_{mp}\) and \(V_{oc}/V_{mp}\). The way in which these ratios vary with fill-factor is also shown in the figure. Finally, the individual points shown are values of \(I_{sc}/I_{mp}\) and \(V_{oc}/V_{mp}\) for modules produced by four different manufacturers.

Figure 2 shows how well (1) represents the current-voltage characteristics of two of these four modules given values for \(I_{mp}, V_{mp}\) and \(f_f\).

A POWER LOSS IN A SERIES STRING

For \(N\) cells connected in series, we add the voltage differentials across each cell operating at a common current to obtain \(V\), the voltage across the entire string. Equation (1) yields:

$$\frac{V}{V_{mp}} = \sum_{r=1}^{N} (V_r/V_{mp}) = \frac{1}{c} \ln \left[ \frac{(\alpha_1 - I_{mp} I_{mp}) (\alpha_2 - I_{mp} I_{mp}) \ldots (\alpha_N - I_{mp} I_{mp})}{\beta_1 \beta_2 \ldots \beta_N} \right] \quad I < \alpha_1$$

(10)

and obtain the following two relationships for determining the series string max-power current, \(I_{mp}\), and max-power voltage, \(V_{mp}\):

$$\frac{V_{mp}}{V_{oc}} = \frac{1}{c} \sum_{r=1}^{N} \frac{(I_{mp} I_{mp})}{(\alpha_r - I_{mp} I_{mp})}$$

$$\exp \left\{ c \left[ V_{mp} - \sum_{r=1}^{N} \frac{V_{mp}(r)}{V_{mp}} \right] / V_{mp} \right\} = \left( \frac{\alpha_1 - I_{mp} I_{mp}}{\alpha_1 - I_{mp} I_{mp}} \right) \left( \frac{\alpha_2 - I_{mp} I_{mp}}{\alpha_2 - I_{mp} I_{mp}} \right) \ldots$$

To avoid getting mired down in cumbersome notation we define unit cell and series string, nondimensional, max-power currents and voltages as follows:
Power loss in photovoltaic arrays

For the rth unit cell:
\[ i_r = I_{mp}(r)/I_{mp} \]
\[ v_r = V_{mp}(r)/V_{mp} \]

For the series string:
\[ I = I_m/I_{mp} \]
\[ V = V_m/V_{mp} \]

The previous two equations may then be written:
\[ V = \frac{1}{c} \sum_{r=1}^{N} \frac{I}{(x_r - I)} \]
\[ \exp \left[ c \left( V - \sum_{r=1}^{N} v_r \right) \right] = \frac{(a_2 - I)}{(a_2 - I_2)} \ldots \frac{(a_N - I)}{(a_N - I_N)} \]  
\[ = \frac{I_{mp} V_{mp} N}{1 + \epsilon \left( \frac{1}{N} \sum_{r=1}^{N} \eta_r + \frac{1}{N} \sum_{r=1}^{N} \xi_r \right)} + \epsilon^2 \left( \frac{1}{N} \sum_{r=1}^{N} \eta_\epsilon \right) \]  

We now assume that the cell max-power currents and voltages may be represented by:
\[ i_r = (1 + \epsilon \eta_r) \]
\[ v_r = (1 + \epsilon \xi_r) \]

where \( \epsilon \) is some small number, and \( \eta_r \) and \( \xi_r \) are non-dimensional measures of variations in cell max-power current and max-power voltage, both of the order of one. If we take \( I_{mp} \) and \( V_{mp} \) as mean values for our universe of cells, then the means of \( \eta_r \) and \( \xi_r \) are zero.

Seeking estimates of the series network max-power current and voltage, which are correct to \( O(\epsilon^2) \), we set:
\[ I = 1 + \epsilon \eta_1 + \epsilon^2 \eta_2 \]
\[ V = N(1 + \epsilon \eta_1 + \epsilon^2 \eta_3). \]

Substitution of (13)–(16) into (11) and (12), expanding and balancing terms of like order in \( \epsilon \), yields:
\[ x_1 = \frac{1}{N} \sum_{r=1}^{N} \xi_r \]
\[ y_1 = \frac{1}{N} \sum_{r=1}^{N} \eta_r \]
\[ x_2 = \frac{c}{2(2+c)} \left[ \left( \frac{x_1}{N} \right)^2 \eta_r - \frac{1}{N} \sum_{r=1}^{N} \eta_r \right] \]
\[ y_2 = \frac{c(1+c)}{2(2+c)} \left[ \left( \frac{x_1}{N} \right)^2 \xi_r - \frac{1}{N} \sum_{r=1}^{N} \xi_r \right] \]
\[ = \frac{\epsilon^2}{N} \sum_{r=1}^{N} \eta_r \]
\[ = \frac{\epsilon^2}{N} \sum_{r=1}^{N} \xi_r \]

To carry out this expansion process, it was necessary to require that \( \epsilon \) be small with respect to one. We expect, then, that our estimates obtained for \( I \) and \( V \) by this method will be good estimates only if variations in cell max-power current and voltages, represented by \( \eta \) and \( \xi \), are limited to a few percent of their mean values.

An alternative approach will be pursued to accommodate larger variations in \( I_{mp} \) and \( V_{mp} \).

We take as a nondimensional measure of power loss due to mismatch:
\[ \Delta P = (P_{\text{max}} - P_{\text{mp}})/(I_{mp} V_{mp} N) \]  

where \( P_{\text{max}} \) is the sum of the max-power outputs of the cells operating independently and \( P_{\text{mp}} \) is the product \( I_m V_m \).

We now assume that the cell max-power currents and voltages may be represented by:
\[ i_r = (1 + \epsilon \eta_r) \]
\[ v_r = (1 + \epsilon \xi_r) \]

where \( \epsilon \) is some small number, and \( \eta_r \) and \( \xi_r \) are non-dimensional measures of variations in cell max-power current and max-power voltage, both of the order of one. If we take \( I_{mp} \) and \( V_{mp} \) as mean values for our universe of cells, then the means of \( \eta_r \) and \( \xi_r \) are zero.

Seeking estimates of the series network max-power current and voltage, which are correct to \( O(\epsilon^2) \), we set:
\[ I = 1 + \epsilon \eta_1 + \epsilon^2 \eta_2 \]
\[ V = N(1 + \epsilon \eta_1 + \epsilon^2 \eta_3). \]

Substitution of (13)–(16) into (11) and (12), expanding and balancing terms of like order in \( \epsilon \), yields:
\[ x_1 = \frac{1}{N} \sum_{r=1}^{N} \xi_r \]
\[ y_1 = \frac{1}{N} \sum_{r=1}^{N} \eta_r \]
\[ x_2 = \frac{c}{2(2+c)} \left[ \left( \frac{x_1}{N} \right)^2 \eta_r - \frac{1}{N} \sum_{r=1}^{N} \eta_r \right] \]
\[ y_2 = \frac{c(1+c)}{2(2+c)} \left[ \left( \frac{x_1}{N} \right)^2 \xi_r - \frac{1}{N} \sum_{r=1}^{N} \xi_r \right] \]
\[ = \frac{\epsilon^2}{N} \sum_{r=1}^{N} \eta_r \]
\[ = \frac{\epsilon^2}{N} \sum_{r=1}^{N} \xi_r \]

To carry out this expansion process, it was necessary to require that \( \epsilon \) be small with respect to one. We expect, then, that our estimates obtained for \( I \) and \( V \) by this method will be good estimates only if variations in cell max-power current and voltages, represented by \( \eta \) and \( \xi \), are limited to a few percent of their mean values.

An alternative approach will be pursued to accommodate larger variations in \( I_{mp} \) and \( V_{mp} \).

We take as a nondimensional measure of power loss due to mismatch:
\[ \Delta P = (P_{\text{max}} - P_{\text{mp}})/(I_{mp} V_{mp} N) \]  

where \( P_{\text{max}} \) is the sum of the max-power outputs of the cells operating independently and \( P_{\text{mp}} \) is the product \( I_m V_m \).

We now assume that the cell max-power currents and voltages may be represented by:
\[ i_r = (1 + \epsilon \eta_r) \]
\[ v_r = (1 + \epsilon \xi_r) \]

where \( \epsilon \) is some small number, and \( \eta_r \) and \( \xi_r \) are non-dimensional measures of variations in cell max-power current and max-power voltage, both of the order of one. If we take \( I_{mp} \) and \( V_{mp} \) as mean values for our universe of cells, then the means of \( \eta_r \) and \( \xi_r \) are zero.

Seeking estimates of the series network max-power current and voltage, which are correct to \( O(\epsilon^2) \), we set:
\[ I = 1 + \epsilon \eta_1 + \epsilon^2 \eta_2 \]
\[ V = N(1 + \epsilon \eta_1 + \epsilon^2 \eta_3). \]

Substitution of (13)–(16) into (11) and (12), expanding and balancing terms of like order in \( \epsilon \), yields:
\[ x_1 = \frac{1}{N} \sum_{r=1}^{N} \xi_r \]
\[ y_1 = \frac{1}{N} \sum_{r=1}^{N} \eta_r \]
\[ x_2 = \frac{c}{2(2+c)} \left[ \left( \frac{x_1}{N} \right)^2 \eta_r - \frac{1}{N} \sum_{r=1}^{N} \eta_r \right] \]
\[ y_2 = \frac{c(1+c)}{2(2+c)} \left[ \left( \frac{x_1}{N} \right)^2 \xi_r - \frac{1}{N} \sum_{r=1}^{N} \xi_r \right] \]
\[ = \frac{\epsilon^2}{N} \sum_{r=1}^{N} \eta_r \]
\[ = \frac{\epsilon^2}{N} \sum_{r=1}^{N} \xi_r \]

Thus
\[ \Delta P = \epsilon^2 \left( \frac{c + 2}{2} \left[ \left( \frac{1}{N} \sum_{r=1}^{N} \eta_r \right)^2 - \frac{1}{N} \sum_{r=1}^{N} \xi_r^2 \right] \right) \]

From (17)–(19) we find:
\[ P_{mp} = N \left[ 1 + \epsilon \left( \frac{1}{N} \sum_{r=1}^{N} \eta_r + \frac{1}{N} \sum_{r=1}^{N} \xi_r \right) \right] \]
\[ + \epsilon^2 \left( \frac{1}{N} \sum_{r=1}^{N} \eta_\epsilon \right) \]

Thus
\[ \Delta P = \epsilon^2 \left( \frac{c + 2}{2} \left[ \left( \frac{1}{N} \sum_{r=1}^{N} \eta_r \right)^2 - \frac{1}{N} \sum_{r=1}^{N} \xi_r^2 \right] \right) \]

If we assume, for the moment, that the cell max-power current is uncorrelated with the max-power voltage, then the expected value:
\[ E[\xi_r, \eta_r] = 0 \quad r, s = 1, N. \]

Furthermore, letting \( \sigma^2 \) be the variance of \( \epsilon \eta \),
\[ \sigma^2 = E[\epsilon^2 \eta^2] \]
then
\[ E \left[ \left( \frac{\epsilon}{N} \sum_{r=1}^{N} \eta_r \right)^2 \right] = \sigma^2 / N \]

represents the expected values of the variance of an \( N \) member sample estimate of the mean of \( \eta \). This relationship holds exactly if the \( \eta \) are normally distributed, otherwise it is an approximation that increases in accuracy with increasing \( N \). Finally, noting that:
\[ E[\eta] = 0 \quad \text{and} \quad E[\xi] = 0 \]
we obtain for estimates of the expected value of maximum power and fractional power loss a series string:

\[ E[P_{\text{mp}}] = N \left[ 1 - \frac{(c + 2)}{2} \sigma_e^2 \left( 1 - \frac{1}{N} \right) \right] \]

\[ E[\Delta P] = \frac{(c + 2)}{2} \sigma_e^2 \left( 1 - \frac{1}{N} \right). \]  

(23)

4. POWER LOSS IN A PARALLEL STRING

For \( M \) cells connected in parallel operating at a common voltage, \( V \), the resultant current is the sum of the currents through the individual cells:

\[ I = \sum_{r=1}^{M} I_r. \]

From the requirements:

\[ \left( \frac{dI}{dV} \right)_{V_m} = -\frac{V_m}{V_m} \]

\[ I(V_m) = I_m \]

we may obtain the following two equations for the determination of \( I_m \) and \( V_m \):

\[ I(1 + cV) = cV \sum_{r=1}^{M} \alpha_r \]

\[ I = cV \exp \left[ cV \sum_{r=1}^{M} \beta_r \right] \]

(24)

(25)

where we have expressed the parallel string max-power current and max-power voltage in non-dimensional form.

Again, expressing the cell max-power currents and voltages by (13) and (14) the parallel string max-power current and max-power voltage by:

\[ I = M(1 + cy_1 + e^y_1y_2) \]

\[ V = 1 + ex_1 + e^x_1x_2 \]

carrying out the same expansion procedure, we obtain:

\[ x_1 = \frac{1}{M} \sum_{r=1}^{M} \xi_r \]

\[ y_1 = \frac{1}{M} \sum_{r=1}^{M} \eta_r \]

\[ x_3 = \frac{c - 3c + 4}{2(2 + c)} \left[ \left( \sum_{r=1}^{M} \xi_r \right)^2 - \frac{1}{M} \sum_{r=1}^{M} \xi_r \right]^2 \]

\[ y_2 = \frac{c^2 - 3c + 4}{2(2 + c)} \left[ \left( \sum_{r=1}^{M} \xi_r \right)^2 - \frac{1}{M} \sum_{r=1}^{M} \xi_r \right]^2. \]

(26)

(27)

(28)

(29)

The maximum power available from our parallel network is then:

\[ P_{\text{mp}} = \sum_{r=1}^{M} \left[ 1 + e \left( \frac{1}{M} \sum_{r=1}^{M} \eta_r \right) + e \left( \frac{1}{M} \sum_{r=1}^{M} \xi_r \right) \right] + e^2 \left[ \frac{(c + 2)}{2} \left( \sum_{r=1}^{M} \xi_r \right)^2 - \frac{1}{M} \sum_{r=1}^{M} \xi_r \right]^2 \]

and the fractional power loss is given by:

\[ \Delta P = \frac{(c + 2)}{2} \left[ \frac{1}{M} \left( \sum_{r=1}^{M} \xi_r \right)^2 - \frac{1}{M} \sum_{r=1}^{M} \xi_r \right]^2 \]

\[ + \left[ \left( \sum_{r=1}^{M} \xi_r \right) - \left( \sum_{r=1}^{M} \xi_r \right) \right] \left( \sum_{r=1}^{M} \xi_r \right) \]

(30)

Taking expected values, again assuming that the cell max-power currents and voltages are uncorrelated, yields:

\[ E[P_{\text{mp}}] = M \left[ 1 - \frac{(c + 2)}{2} \sigma_e^2 \left( 1 - \frac{1}{M} \right) \right] \]

\[ E[\Delta P] = \frac{(c + 2)}{2} \sigma_e^2 \left( 1 - \frac{1}{M} \right) \]

where \( \sigma_e^2 \) is the variance of the variations in max-power voltage of the cells:

\[ \sigma_e^2 = E[(e\xi)^2]. \]

5. MORE COMPLEX NETWORKS

Having obtained estimates of max-power current and max-power voltage for either a series string or parallel string of cells, estimates correct to \( O(\epsilon^2) \), we may, in turn, synthesize series and parallel strings of those basic networks and derive estimates of the power loss due to mismatch for more complex arrays, again correct to \( O(\epsilon^2) \). For example, consider a network of \( M \) strings in parallel, each string made up of \( L \) cells in series. Considering each series substring as a unit cell we can identify variations in its max-power current, \( \eta_m \), and max-power voltage, \( \xi_m \), with the results obtained for estimates of \( \eta_m \) and \( \xi_m \) for a series string of \( L \) cells by the methods used in Section 3. We write:

\[ \eta_m = y_1(\eta_m, \xi_m) + e y_2(\eta_m, \xi_m) \]

\[ \xi_m = x_1(\eta_m, \xi_m) + e x_2(\eta_m, \xi_m) \]

where \( \eta_m \) and \( \xi_m \) represent variations in max-power current and max-power voltage of the cell located in the \( r \)th parallel string, at the \( s \)th position and \( x_1 \), \( x_2 \), \( y_1 \), and \( y_2 \) are given by (17)–(19), replacing \( N \) by \( L \) and \( \eta_m \), \( \xi_m \), \( \eta_s \), and \( \xi_s \).

Equations (26)–(29) then provide estimates of \( I_m \) and \( V_m \) for a series string of \( L \) cells by the methods used in Section 3. We write:

\[ E[P_{\text{mp}}] = M \left[ 1 - \frac{(c + 2)}{2} \sigma_e^2 \left( 1 - \frac{1}{M} \right) \right] \]

\[ E[\Delta P] = \frac{(c + 2)}{2} \sigma_e^2 \left( 1 - \frac{1}{M} \right) \]

(31)

In deriving (31) we have neglected terms of order \( \epsilon^3 \) with respect to \( I \), again assuming that variations in cell max-power currents and voltages are uncorrelated, and estimated:

\[ E[\Delta P] = \frac{(c + 2)}{2} \sigma_e^2 \left( 1 - \frac{1}{M} \right) \]

\[ + \left[ \sum_{r=1}^{M} \xi_r \right] \left[ \left( \frac{1}{M} \sum_{r=1}^{M} \xi_r \right)^2 - \frac{1}{M} \sum_{r=1}^{M} \xi_r \right]^2 \]
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by

\[-\frac{\alpha_c^2}{L} \left( 1 - \frac{1}{M} \right) \]

Note that (31) reduces to (30) when \( L \) is set to 1, and to (23) when \( M \) is set to 1, \( L \) replacing \( N \) in (23). This process can be continued indefinitely. For example, for \( N \) series blocks of \( M \) parallel substrings of \( L \) cells in series we obtain:

\[ E[\Delta P] = \frac{(c+2)\alpha_c^2}{2} \left( 1 - \frac{1}{T} \right) + \frac{\alpha_c^2}{L} \left( 1 - \frac{1}{M} \right) + \frac{\alpha_c^2}{ML} \left( 1 - \frac{1}{N} \right) \] \hspace{1cm} (32)

If the total number of cells in the network is \( T \), for example,

\[ LMN = T \] \hspace{1cm} (33)

then, eliminating \( L \) in favor of \( T \), we obtain:

\[ E[\Delta P] = \frac{(c+2)\alpha_c^2}{2} \left( 1 - \frac{1}{T} \right) \left( \sigma_c^2 - \sigma^2 \right) \left( \frac{N}{T} \right) (T-M-1) \] \hspace{1cm} (34)

6. EXAMPLE: POWER LOSS IN A SERIES STRING

We consider a problem treated in [2]; determine the power loss due to mismatch of a series string of cells exhibiting a gaussian probability density of cell max-power currents, but no variations in cell max-power voltage. The max-power currents have a mean of:

\[ I_{mp} = 39.2 \text{ ma/cm}^2 \]

and standard deviation:

\[ \sigma = 2.86 \text{ ma/cm}^2. \]

In terms of our nondimensional variables we have:

\[ \sigma_c^2 = (\sigma/\bar{I}_{mp})^2 = 5.32 \times 10^{-3}, \]

i.e. a standard deviation in max-power currents of, roughly, 7 per cent of their mean. The max-power voltage of the cell is given as:

\[ V_{mp} = \bar{V}_{mp} = 0.4174 \text{ volts} \]

while the cell I-V curve has a fill-factor estimated to be equal to 0.670. Equation (9) yields \( c = 6.83 \).

From the results of the analysis presented in Section 3, we have, for \( N \) cells in series:

\[ E[P_{max}] = 16.36 \times 10^{-3} \text{ W/cm}^2 \]

\[ E[P_{mp}] = 15.9 \times 10^{-3} \text{ W/cm}^2 \]

where we have neglected \( 1/N \) with respect to one. The fractional power loss due to mismatch is:

\[ E[\Delta P] = 2.35 \text{ per cent.} \]

These replicate the result presented in [2], results obtained by means of numerical evaluation of the location of the series string max-power point. (A fractional loss of 3 per cent is given in [2] whereas \( E[P_{max}] \) and \( E[P_{mp}] \) match to three significant figures. The loss of accuracy due to the differencing of two similar numbers accounts for the apparent discrepancy in estimates of the fractional power loss due to mismatch.)

7. EXAMPLE: A MORE COMPLEX NETWORK

We consider a network of \( N \) series blocks of \( M \) parallel substrings each of the latter having \( L \) cells in series. Three examples of this type of array have been analyzed in [3] using Monte Carlo simulation techniques; a random selection of cells, followed by a numerical synthesis of the network I-V curve and computation of \( L_m \) and \( V_m \).

Individual cells were assumed to have the same max-power voltage, while the I-V curve was taken to have a fill-factor of 0.70. Equation (9) yields,

\[ c = 8.13. \]

The three cases differed in assumed probability densities of cell short-circuit current, which by means of (7) can be translated directly into densities of max-power currents. Probability densities so calculated are shown in Fig. 3. The corresponding nondimensional variances are:

\[ \sigma_c^2_{case 1} = 0.0482^2 \]

\[ \sigma_c^2_{case 2} = 0.09483^2 \]

\[ \sigma_c^2_{case 3} = 0.08772^2. \]

The expected value of the fractional decrease in maximum output power may be obtained from (34) setting \( \sigma_c \) to zero and \( c = 8.13 \). Taking the total number of cells to be 192, as in [3], we have:

\[ E[\Delta P] = 5.065 \sigma_c^2(0.995 - (5.21 \times 10^{-3})(M - 1)). \] \hspace{1cm} (35)

Figures 4-6 show both \( E[\Delta P] \) as obtained from the above and as presented in [3]. Cases 1 and 2 compare well while results for case 3 suggest that our estimation procedure is in error.

The fault lies in applying an approximate technique outside its domain of validity. If we take the half-range of the probability density as a measure of \( \sigma_c \), we obtain, for the weighted variation in max-power current, \( ec \), 0.80 for the case 1, 1.58 for case 2, and 1.71 for case 3. Recall that this estimation procedure is intended to apply only when \( ec \) is less than one. It is thus not surprising that our results for case 3 do not compare well with results obtained by numerical methods which place no restrictions on the magnitude of \( ec \). Note, however, that case 2 also violates this "smallness" criterion, suggesting that the shape of the probability density, or equivalently, higher moments of the probability density, may significantly affect the power loss due to mismatch. These features are studied further in what follows where we relax this constraint on \( ec \).

D-5
We now allow variations in cell max-power current and voltage to be larger relative to their mean values such that the product \(ce\) may now be of order one. In fact, we define \(e\) by:
\[
e = c^{-1},
\]
and again seek estimates for \(I\) and \(V\), the nondimensional max-power current and voltage of a series string of \(N\) cells, by means of an expansion of (11) and (12) on \(e\). In this case, however, we expand all currents about the minimum short-circuit current value available in the set of cells at our disposal. We let, as before:
\[
p_r = 1 + \epsilon \xi_r,
\]
\[
V = N(1 + \epsilon x_1 + \epsilon^2 x_2)
\]
but now put:
\[
\eta_r = 1 + \epsilon \eta_r,
\]
\[
\epsilon V = N(1 + \epsilon x_1 + \epsilon^2 x_2)
\]
(37)

where \(\alpha_1\) is the minimum, nondimensional, short-circuit, cell current available.

To obtain expected values for the string maximum output power and percent power loss due to mismatch, we need some measures of the statistical variations in cell max-power current and voltage, \(\eta\) and \(\xi\). We assume that we are given frequency distributions of these characteristics. In particular, we assume that we randomly and independently draw each of the \(N\) cells in the series string from a set of cells that has been sorted into \(T\) subsets. All cells within the \(r\)th subset have identical max-power currents, \(i_r\), or since short-circuit current is, in this study, directly proportional to max-power current, identical short-circuit currents, \(\alpha_r\). We arrange and label the subsets so that the first subset contains all cells with the minimum short-circuit current, \(\alpha_1\); the second, cells with the next largest short-circuit current, \(\alpha_2\), etc. Thus
\[
\alpha_1 < \alpha_2 < \alpha_3 \cdots < \alpha_T.
\]
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Fig. 6. Power loss due to mismatch, case 3.

Of the $N$ cells selected, let $N_a$ be the number of cells chosen that have a short-circuit current of $a_m$, the minimum short-circuit current in the lot of $N$ cells. Let $N_{a_1},$ be the number of cells chosen from the subset having the next largest short-circuit current, etc. Clearly,

$$\sum_{i=1}^{T} N_i = N.$$  

Note that the minimum $a$ selected, $a_m$, need not be the minimum available, $a_1$; that is, $a_m > a_1$, and that, while $N_a$, cannot by definition be zero, the number of cells chosen from the $m + k$ subset, $1 < k < T - m$, may be zero.

With these preliminaries and setting $c = e^{-1}$, expansion of (11) and (12) upon $e$ yields, after balancing powers of $e$:

$$y_1 = \eta_m$$

$$y_2 = -N_m/eN$$

$$x_1 = \frac{1}{N} \sum_{i=1}^{N} \xi + \frac{1}{N} \ln (1 + eN) + \frac{N_m}{N} \ln \left( \frac{N_m}{N} \right)$$

$$+ \frac{N_{a_1}}{N} \ln (\eta_{a_1} - \eta_m) + \cdots + \frac{N_{T}}{N} \ln (\eta_T - \eta_m). \quad (39)$$

We choose not to go on and determine $x_2; x_1$ will provide an adequate estimate of the string max-power voltage. Note that we expect the product $eN$ to be of order one. It is also necessary that there exist an $\eta$ of order one for these results to apply.

The string max-power current and max-power voltage are then:

$$I = a_m [1 + \epsilon N_m]$$

$$V = N \left\{ 1 + e \left[ \frac{1}{N} \sum_{i=1}^{N} \xi + \frac{1}{N} \ln (1 + eN) + \frac{N_m}{N} \ln \left( \frac{N_m}{N} \right) \right. \right.$$  

$$+ \left. \frac{\sum_{j=m+1}^{T} N_j \ln (\eta_j - \eta_m)}{N} \right\}. \quad (41)$$

Now $a_m (1 + eN_m)$ is just $a_m$, the minimum short-circuit current selected. Furthermore, by (4):

$$a_l = i_1 (1 + \epsilon) + O(\epsilon^2).$$

Thus, we may write:

$$I = a_m \left[ 1 - \epsilon \frac{N_m}{N} \right] = i_m \left[ 1 + \epsilon \left( 1 - \frac{N_m}{N} \right) \right]. \quad (42)$$

Similarly,

$$\eta_l = (a_l - a_1) \frac{\epsilon a_1}{(i_1 - i_m)} \quad (43)$$

where $i_m$ is the minimum max-power current selected while $i_l$ is the minimum max-power current available.

The expected value of the maximum output power for the series string of $N$ cells is then:

$$E[P_{max}] = NE[i_m] \left\{ 1 + e \left[ \frac{E[\xi_1] - E[i_m]}{E[i_m]} \right] \right\},$$

where we have dropped the subscript 1 on $\xi$ for convenience, and again assumed that the $\xi$ and $\eta$ are uncorrelated. One consequence of this, and the fact that $E[\xi] = 0$ by definition, is that variations of order $\epsilon$ in cell max-power voltage have a null effect on the expected value of max-output power of the series string, again correct to order $\epsilon$.

The expected value of the percent power loss due to mismatch is:

$$E[\Delta P] = 1 - E[i_m] \left\{ 1 + e \left[ \frac{E[\xi_1] - E[i_m]}{E[i_m]} \right] \right\}. \quad (44)$$

To determine the expected values appearing in the foregoing, we turn to the statistics of $i$, the nondimensional, max-power cell current. Let $p_r$ be the probability of choosing a cell from the $r$th subset of cells; that subset of cells with a max-power current of $i_r; r = 1, T$. Then the probability of a series string of $N$ cells containing $N_r$ cells each with the (minimum available) max-power current $i_r; N_1$ cells each with max-power current, $i_2, \ldots, N_T$ cells each with max-power current, $i_T$ (the maximum available), is given by the multinomial density function (1):

$$p(N) = \frac{N!}{N_1! \cdots N_T!} p_1^{N_1} p_2^{N_2} \cdots p_T^{N_T}$$

where

$$\sum_{r=1}^{T} N_r = N$$

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The expected value of the minimum max-power current selected, \( E[i_m] \), is then:

\[
E[i_m] = \sum_{\tilde{N}} i_m(\tilde{N})p(\tilde{N})
\]

where

\[
i_m(N) = \begin{cases} 
  i_1 & N_1 \neq 0 \\
  i_2 & N_1 = 0; \; N_2 \neq 0 \\
  \vdots \\
  i_r & N_1 = N_2 = \cdots = N_{r-1} = 0; \; N_r = N \neq 0.
\end{cases}
\]

We may thus write:

\[
E[i_m] = i_1 \sum_{\tilde{N}_1=0}^{N_1} p(\tilde{N}) + i_2 \sum_{\tilde{N}_2=0}^{N_2} p(\tilde{N}) + \cdots + i_r \sum_{\tilde{N}_r=0}^{N_r} p(\tilde{N})
\]

where each summation may be shown to be equal to:

\[
\sum_{\tilde{N}_r=0}^{N_r} p(\tilde{N}) = \left[ 1 - (p_1 + p_2 + \cdots + p_{r-1})^N \right] - \left[ 1 - (p_1 + p_2 + \cdots + p_r) \right]^{N-1} (p_{r-1} = 0 \; \text{if} \; j = 1)
\]

and represents the probability that the minimum max-power current appearing in the series string will have been selected from the \( j \)th subset.

Evaluation of \( E[i_m \cdot x] \) follows in much the same way except that an approximation is required to take care of the term \( (N_m/N) \ln (N_m/N) \), an approximation that loses its grip as \( N \) gets small. That is:

\[
E[i_m \cdot x] = \sum_{\tilde{N}} i_m(\tilde{N})x(\tilde{N})p(\tilde{N})
\]

where \( i_m(N) \) is given above and from (39)

\[
x(\tilde{N}) = \frac{1}{N} \ln (1 + \epsilon N) + \frac{N_m}{N} \ln \left( \frac{N_m}{N} \right)
\]

\[
+ \sum_{j=1}^{M} \frac{N_j}{N} \ln (\eta_j - \eta_m).
\]

We can show approximately that:

\[
\sum_{\tilde{N}_r=0}^{N_r} N_m \ln \left( \frac{N_m}{N} \right) p(\tilde{N}) = p_m \ln \left( \frac{p_m + (1 - p_m)}{N} \right) \left[ 1 - (p_1 + p_2 + \cdots + p_{r-1}) \right]^{N-1}
\]

where again, \( p_m = 0 \) if \( m = 1 \). The remaining terms may be shown to be:

\[
\sum_{N_m=0}^{N_r} p(\tilde{N}) \left\{ \sum_{j=1}^{M} \frac{N_j}{N} \ln (\eta_j - \eta_m) \right\}
\]

\[
= \left[ 1 - (p_1 + p_2 + \cdots + p_{r-1}) \right]^{N-1}
\]

\[
- \left[ 1 - (p_1 + p_2 + \cdots + p_r) \right]^{N-1}
\]

Altogether, we have then:

\[
E[i_m \cdot x] = \frac{1}{N} \ln (1 + \epsilon N) E[i_m]
\]

\[
+ \sum_{j=1}^{M} i_m \ln \left[ (p_m + (1 - p_m)) \right]
\]

\[
\times \left[ 1 - (p_1 + p_2 + \cdots + p_{r-1}) \right]^{N-1}
\]

\[
+ i_m \ln \left\{ 1 - (p_1 + p_2 + \cdots + p_r) \right\}^{N-1}
\]

\[
\times \sum_{j=1}^{M} p_j \ln (\eta_j - \eta_m)
\]

By the same technique, we obtain without any approximation:

\[
E[i_m \cdot x] = \sum_{\tilde{N}} i_m(\tilde{N})x(\tilde{N})p(\tilde{N})
\]

Application of these formulae to particular examples is deferred to Section 10.

9. POWER LOSS IN A PARALLEL STRING \([c = 0c^{-1}]\)

For \( M \) cells in parallel with \( c = 1 \), we obtain, by the same expansion process, that the parallel string max-power current is equal to the sum of the \( M \) individual cell max-power currents, that is:

\[
I = \sum_{i=1}^{M} \xi_i = M \left( 1 + \sum_{i=1}^{M} \xi_i \right)
\]

where now we have considered variations in cell max-power currents about their mean value as in (13).

The max-power voltage is found to be:

\[
V = 1 + \epsilon \ln \left\{ \sum_{i=1}^{M} \exp [- \xi_i] \right\}
\]

where \( \xi_i \) is defined by (14). These results are correct to \( O(\epsilon) \). Note that if we formally expand the exponential, we obtain:

\[
V = 1 + \epsilon \left\{ \sum_{i=1}^{M} \xi_i - \frac{1}{2} \left[ \sum_{i=1}^{M} \xi_i^2 - \left( \frac{1}{M} \sum \xi_i \right)^2 \right] \right\}
\]

We expect this to yield a valid approximation only if \( \xi < 1 \). Putting \( c = \epsilon^{-1} \) in (28) and retaining terms of order \( \epsilon \), in the definition of \( V \) presented in Section 4, yields this same expression for max-power voltage indicating a correspondence of the two estimation pro-
Power loss in photovoltaic arrays

Fig. 7. Probability densities for minimum max-power current series string of $L$ cells, case 3.

Expected values of $I$ and $V$, of the parallel string maximum output power and of the percent loss in power due to mismatch may be written down as before. The presence of the logarithm of a sum of exponentials appearing in the expression for max-power voltage complicates evaluation of these expected values.

10. EXAMPLE: $e = 0.03^{-1}$, A COMPLEX ARRAY

We return to the examples treated in Section 7. There we were confronted with significant discrepancies between our results and results obtained by other means, particularly for case 3. For that case we have calculated, by means of (45), the probability density for $I_m$, the minimum max-power current found in a series string of $L$ cells, for various values of $L$. The results for $L = 96, 24, 6$ cells are shown in Fig. 7. The expected values required for the estimation of the percent power loss due to mismatch among $L$ cells in series have been computed by means of the relationships derived in Section 8 and are given in Table 1 along with $\Delta P$.

To proceed further, to obtain results for the power loss due to mismatch for a series string of $N$ blocks of parallel strings, each parallel string composed of $M$ series strings of $L$ cells, we will apply the results obtained earlier for the case in which variations in max-power currents and voltages are small relative to their means; that is, for the case in which $e < 1$, where now $e$ is to be understood as a measure of these variations.

Considering those cases when $L$ is not small and referring to Fig. 7, we note that variations in the substring’s max-power current as measured by the half-range appear sufficiently small to justify this approach. As $L$ gets smaller, as the probability density function for the max-power current spreads out, we might anticipate some difficulty. Estimates of the variance of $x$, variations in the substring’s max-power voltage appear in Table 1 and permit the same argument: for $L$ not small, we expect variations in max-power voltage of our unit substrings to meet the criterion, $e < 1$.

We thus consider the problem of $M$ unit “cells” arranged in parallel and $N$ of those parallel blocks connected in series where each unit cell (in reality a series string of $L$ cells) is drawn from a set of units, each having a max-power current, $I_{mp}$, and max-power voltage, $V_{mp}$, that can be expressed in terms of their mean values, $I_{mp}$ and $V_{mp}$, and variations from those means, $\eta$ and $\xi$; i.e.

\[
I_m = I_{mp}(1 + \eta) \\
V_m = V_{mp}(1 + \xi).
\]

Table 1. Parameters defining power loss in a series string of $L$ cells, case 3

<table>
<thead>
<tr>
<th>$L$</th>
<th>$E[I_m]$</th>
<th>$E[I_{mx}]$</th>
<th>$E[I_{m}^2/L]$</th>
<th>$E[\Delta P]$</th>
<th>$E[%]$</th>
<th>$E[x]$</th>
<th>$\sigma^2_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>96</td>
<td>0.772</td>
<td>0.549</td>
<td>0.031</td>
<td>6.9</td>
<td>0.712</td>
<td>0.0032</td>
<td></td>
</tr>
<tr>
<td>72</td>
<td>0.773</td>
<td>0.554</td>
<td>0.032</td>
<td>6.7</td>
<td>0.717</td>
<td>0.0088</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>0.782</td>
<td>0.526</td>
<td>0.040</td>
<td>6.2</td>
<td>0.676</td>
<td>0.0258</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0.803</td>
<td>0.439</td>
<td>0.062</td>
<td>5.2</td>
<td>0.558</td>
<td>0.0582</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.835</td>
<td>0.280</td>
<td>0.113</td>
<td>4.1</td>
<td>0.355</td>
<td>0.1022</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.878</td>
<td>0.064</td>
<td>0.223</td>
<td>3.9</td>
<td>0.099</td>
<td>0.1180</td>
<td></td>
</tr>
</tbody>
</table>
But $I_{mp}$ and $V_{mp}$ are also the max-power currents and voltages, respectively, of a series string of $L$ cells; i.e.

$$I_{mp} = I_{mp} L \left[ 1 + \epsilon \left( 1 - \frac{I_{lm}}{L} \right) \right]$$

$$V_{mp} = V_{mp} L \left[ 1 + \epsilon x \right]$$

where $\epsilon = c^{-1}$, $I_{lm}$ is the minimum max-power current in the $L$ cell series substring, and $x$ is as defined in Section 8.

From these relationships we find that:

$$I'_{mp} = I_{mp} E[I_{lm}] \left[ 1 + \epsilon \left( 1 - \frac{E[I_{lm}]}{E[I_{lm}]} \right) \right]$$

$$V'_{mp} = V_{mp} L \left[ 1 + \epsilon E[x] \right]$$

and the variances of $\eta$ and $\xi$ are:

$$\sigma_\eta^2 = \sigma_\zeta^2 \left( \frac{E[I_{lm}]}{E[I_{lm}]} \right)^2$$

$$\sigma_\xi^2 = \epsilon^2 \sigma_\zeta^2.$$

The expected value of the sum of the individual maximum powers of the cells is, as before,

$$E[P_{max}] = LN_{mp} V_{mp}.$$

While the expected value of the maximum power available from the network of $M$ units in parallel and $N$ of these in series, $E[P_{mp}]$, is:

$$E[P_{mp}] = MN_{mp} V_{mp} \left[ 1 - \frac{c + 2}{2} \right]$$

$$\times \left[ \frac{\sigma_\zeta^2}{M} \left( 1 - \frac{1}{N} \right) + \sigma_\xi^2 \left( 1 - \frac{1}{M} \right) \right].$$

The fractional power loss due to mismatch is then:

$$\Delta P = \frac{E[P_{max}] - E[P_{mp}]}{MN_{mp} V_{mp}}$$

$$= 1 - E[I_{lm}] \left[ 1 + \epsilon \left( 1 + E[x] - \frac{E[I_{lm}]}{E[I_{lm}]} \right) \right]$$

$$\times \left[ 1 - \frac{c + 2}{2} \right]$$

$$\times \left[ \frac{\sigma_\zeta^2}{M} \left( 1 - \frac{1}{N} \right) + \epsilon^2 \sigma_\zeta^2 \left( 1 - \frac{1}{M} \right) \right].$$

Evaluation of $\Delta P$ for varies combinations of $M$ and $N$ where, as before, the total number of cells is taken as $192 = LMN$, yields now a quite accurate estimation of power loss due to mismatch for case 3, at least, as measured against the results presented in [3]. The dashed curves in Fig. 8 have been obtained in this way. Clearly, we have improved our estimate of power loss due to mismatch.

II. EXAMPLE: POWER LOSS IN A SERIES STRING ($\sigma = 0 \sigma^{-1}$)

Here we explore the effect of varying probability-density-function shape on mismatch loss in a series string of $N$ cells. For the case, $\sigma < 1$, when the weighted variations in cell characteristics are small relative to their mean values, the analysis of Sections 3 and 4 indicates that the mismatch loss is shape independent. The results presented in [3], on the other hand, suggest that losses may become dependent on the shape of the probability density function as the weighted variations in cell max-power current and voltage become larger; as $\sigma \to 1$. We thus investigate mismatch losses for two probability densities governing variation in cell max-power current: (1) a gaussian density truncated at the $1 \pm 3\sigma$ points, and (2) a uniform density of $[2/\sigma(3\sigma)]^{-1}$ over a finite range, $1 \pm 3\sigma$. The max-power voltages are assumed to show no variation from cell to cell.

For a continuous probability density of cell max-power current, $\phi(i)$, the probability density function for the minimum value of max-power current chosen in a sample site of $N$ cells may be shown to be:

$$p(i_{lm}) = N\phi(i)[1 - \Phi(i)]^{N-1} = -\frac{d}{di} [1 - \Phi(i)]^N \tag{47}$$

where $\Phi(i)$ is the probability distribution function:

$$\Phi(i) = \int_{-\infty}^{i} \phi(x) \, dx.$$

The correspondence with the discrete probability density function for minimum current value selected, (44), is to be noted.
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The gaussian density of max-power currents is taken as:

$$\phi(i) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2} \left(\frac{i-\mu}{\sigma}\right)^2\right]$$

where the mean of the nondimensional current values is, by definition, 1. Rather than resorting to numerical integration to determine the expected value of $i_m$, needed in the calculation of $\Delta P$, we apply the results obtained by others for the expected value of the range, $R$, of a sample of $N$ items drawn from a gaussian distribution:

$$E[i_m] = 1 - \frac{1}{2} E[R]$$

or

$$E[i_m] = 1 - \beta(N)\sigma$$

where $\beta$ is a factor depending on $N$.

Calculation of $E[\Delta P]$ also requires an estimation of the series string max-power voltage, or equivalently, $E[i_{mx}]$. Equation (46) yields:

$$E[i_{mx}] = \frac{E[i_m]}{N} \ln \left(\frac{1+\epsilon N}{N}\right) + \frac{(N-1)}{N} E\left[i_m \ln \left(\frac{i_m}{E[i_m]}\right)\right]$$

To effect this integration, we take:

$$p(i_m) = \delta(i_m - E[i_m])$$

where $\delta$ is the Kronecker delta function; a reasonable approximation for $N$ not small. Numerical integration of what remains after this step for selected $\sigma$ and $\beta$ yields the values of $E[i_{mx}]$. Estimates of $\Delta P$ are then obtained from:

$$E(\Delta P) = 1 - E[i_m] \left\{1 + \epsilon \left[1 + \frac{E[i_{mx}]}{E[i_m]} - \frac{1}{N}\right]\right\}$$

and plotted in Fig. 9 as a function of $\sigma$ for $N = 100, 50,$ and $10$. Values of $\Delta P$ for relatively small variations in max-power current, the case when $\epsilon < 1$, are also plotted in the figure. The value of $c$ was taken as 8.13 as before.

![Diagram of power loss in photovoltaic arrays](image-url)
Note that as the variance of the probability density function of cell max-power current increases, as the range of available values increases, the number of cells placed in series become an important factor. As $N$ increases there is an increasing probability of selecting a cell whose max-power current lies closer to the bottom of the range, to the $1-3\sigma$ limit. Figure 7 shows this effect for a given frequency distribution—that of case 3. Note, however, that this effect only becomes significant when the weighted measure of variations in cell max-power current ($\sigma_m$) becomes of order 1; for example, when:

$$\sigma = \frac{1}{2c} = \sigma / 2.$$ 

Also shown in the figure are results obtained for a uniform probability density of cell max-power current over the range $1 \pm \sqrt{3}\sigma$. Here the effect of $N$ is relatively insignificant—reflecting the fact that the probability density function for $I_m$ does not change as dramatically when going from $N = 10$ to $N = 100$ as it does in the case of the truncated gaussian density. Thus two distributions of cell characteristics, both showing the same variance, can give significantly different values for fractional power loss due to mismatch. This explains the differences appearing in Figs. 5 and 6.

12. CONCLUSIONS

We have obtained expressions useful in estimating power loss due to mismatch in series and parallel networks of photovoltaic cells. The most interesting result is the existence of a critical level of variation in cell max-power current; a level above which the fractional power loss due to mismatch in a series string becomes sensitive to the number of cells in series and to the shape of the probability density function of max-power current. In a series string, variations in max-power voltage have a lesser effect on power loss due to mismatch than do variations in max-power current.

In complex arrays, power losses due to mismatch may be estimated by means of the relationships derived in Sections 3 and 4 if variations in current and voltage are sufficiently small. If not, the application of the results obtained for $c = 2^{-1}$ must be used, at least to the first level of cell aggregation. This will yield estimates of the mean and variance of density functions for max-power current and max-power voltage of these new "unit cells". Deviations in max-power current and max-power voltage of these units may now satisfy our smallness criteria. In fact, as the number of cells connected together at this first level of aggregation increases, these deviations will decrease. In the case of connection of $M$ cells in parallel, the density function of the parallel string's max-power current approaches a gaussian density with deviation $\sigma / \sqrt{M}$. Similarly, if $n$ cells are first strung together in series as $N$ increasing the density of the string max-power current, as we have seen, narrows.

REFERENCES