ENVIRONMENTAL HAIL MODEL FOR ASSESSING RISK TO SOLAR COLLECTORS

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LOW-COST SILICON SOLAR ARRAY PROJECT
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RISK TO SOLAR COLLECTORS

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ABSTRACT

Solar photovoltaic arrays deployed in certain areas of the U.S. will be subject to damaging hailstones. Hailstones up to five inches in diameter have caused damage to glass panes in buildings and autos, to asphalt roofs, and to other property and crops.

This report presents the results of a study assessing the probability of solar arrays being struck by hailstones of various sizes as a function of geographic location and service life. The study complements parallel studies of solar array sensitivity to hail damage, the final objective being an estimate of the most cost effective level for solar array hail protection.

A key element of this study involves the generation of a statistical model describing the probability of impact by hailstones of various sizes and estimating the mean time between hits. This model is based on three types of information: the average number of annual hail days at a location; the hailstone size distribution, given that a hailstorm has occurred; and the areal densities of hailstones. Hail statistics in each of these areas are developed from published hail records and private consultation with experts in the field. The general lack of good quality data, particularly on hailstone size distribution and areal densities, necessitates extrapolation of sparse data and leads to large uncertainty bounds on the final results. These uncertainty bounds are analyzed via a sensitivity analysis in which the size and areal density distributions are allowed to vary between maximum and minimum values as determined from available data. This provides a range of values for the study results which are stated in terms of the probability of a given area of solar array being struck by hailstones of a specific size or larger. The results are identified by region, and are defined in terms of the number of annual hail days associated with that region. The results indicate that damage to solar collectors from hailstones may occur in many parts of the country. In the Great Plains region of the United States, where the most damaging hail occurs, it is predicted that solar arrays will be struck by hailstones 1.5 inches or larger as often as every 6 to 8 years. Although these results contain considerable uncertainty, it is concluded that the local hail environment of a proposed site location is an important factor to consider in solar array design and applications. This is particularly true considering the large site-to-site variations even within small regions of the country.
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A. PURPOSE.

A program is underway to develop reliable and economically competitive terrestrial photovoltaic solar arrays. These arrays consist of large areas of silicon solar cells encapsulated by a transparent glass or plastic cover. Such large areas of arrays are vulnerable to degradation caused by weather conditions, solar ultraviolet radiation, pollution, dust, and sand. One potentially destructive element of weather is hail. Hailstones large enough to damage a solar collector of any type occur frequently enough to be of concern in some areas of the country.

The problem which arises is the quantification of risk due to hailstone damage. Unfortunately, very little data are available to model the risk. Values of certain parameters are required in order to perform studies such as determining the cost tradeoffs between providing added protection to solar arrays and assuming the risk of having to replace array modules prior to obsolescence.

Previous work on hail risk has primarily centered on crop damage due to hail. The threshold hailstone size for crop damage is significantly smaller than that which would cause damage to a structure such as a solar array. Therefore, the area of concern in the latter case encompasses the occurrence of events which are rare, but nevertheless, of sufficiently high probability to warrant investigation.

This study involves the development of a hail-risk model using data available only in limited amounts. The model and results reported here are equally applicable to any type of solar collector which is susceptible to impact damage.

B. STUDY APPROACH.

The approach used in the study discussed in this report is based on a statistical model using available data. Initially, a survey of the literature was performed, and contacts made with people in the field to obtain background information and data necessary to perform the study. The report provides background information on the characteristics of hail so that the elements of hail which are important to this study may be seen in the proper context.

A very important aspect of the study is the varied geographical distribution of severe hail activity. This situation dictates that the solution of the hail problem must be specific for each area of the country. A discussion of this aspect appears in the report.

A discussion of the important hail parameters includes the availability and types of data and provides an assessment of the data. The parameters considered include: average number of hailstorms/year,
hailstone size distribution, hailstone areal density, hailstorm duration, and several physical parameters such as terminal velocity. Data for the first three parameters cited above, which were obtained from the literature and through private communication, are plotted and developed. The U.S. is divided into three regions and values of the parameters which are characteristic of each region are used. Values of the parameters required for the hail risk model are obtained from these plots.

After the initial discussion providing information on hail characteristics and data assessment, the report covers the data analysis and risk model development. The model is based on the use of Poisson statistics and a hailstone size distribution. In addition to size, the other parameters used include hailstone number and areal density.

A sensitivity analysis was performed to account for the application of a model using sparse data, with rather large uncertainties. Two parameters were provided as output from the model, the probability of being impacted by a hailstone of a given size, and the mean time between hits (MTBH) for such hailstones. The results of the sensitivity analysis provided a wide range of values for these parameters based on the range of uncertainty of the input parameters.

Finally, some general conclusions based on the results of the analysis performed using the model are presented. These conclusions reflect the severity of the hail problem in different locations for hailstones of various sizes. Recommendations are given for future work in the area of hail risk determination.
A. CHARACTERISTICS OF HAILSTORMS.

1. Past History.

There are recorded occurrences of considerable damage as a result of hailstorms (References 1-7). Hailstones ranging in size from 1.5 inches to 4 inches in diameter are reported to have fallen in a number of areas and caused considerable damage. On the other hand, hailstones described to be as large as grapefruit have occurred without much damage. This occurred in Potter, Nebraska in July 1928 (Reference 3). The areal density is described as 10 stones per town lot, or 10-15 feet apart.

The damage caused by hailstones is a function of their size, wind speed and direction, areal density and location of the storm. Therefore, it is difficult to develop quantitative data from accounts of past hailstorms. However, the reports indicate a wide range of damage including broken glass panes, punctures in asphalt shingled roofs, punctured automobile tops (in the 1920's), demolished green houses, trees stripped bare, crops destroyed, and animals killed.

Another factor which is important is the size of the hail swath which traverses the terrain. Hailstorms normally do not cause widespread damage over a large area. A hailstorm which occurred in Rapid City, South Dakota (Reference 5) was 3 miles wide and 20 to 30 miles long. Although widespread damage occurred in Rapid City, no hail was reported two miles west of the City. A destructive hailstorm which took place in Birmingham, Alabama in 1921 (Reference 6) was reported to be 3 to 8 miles in width and 37 miles long.

The reports cited above also indicate that great quantities of hail fell lasting more than one day in the warm weather. The storm in Birmingham left piles of hail two feet deep in ravines.

To summarize, there are numerous reports of destructive hailstorms having large hailstones associated with them. However, their occurrence is infrequent, the size of the hail swath is limited, and little data are available from which to determine the probability of occurrence of hail of a given size over large areas. The hail networks which have been set up to measure hail are of two types. One type is dependent on results reported by observers and results in data whose accuracy are not easily determined. The second is dependent on devices such as hail pads which are spread out over an area. The latter devices, although providing more accurate data, are too spread out to provide adequate coverage of large hailstone events where the areal density of the stones is smaller than the area of the pads.
2. Association with Thunderstorms.

Most hail, but not all, occurs with thunderstorms. The reverse is not true, i.e., not all thunderstorms produce hail. Hail in the lee of the Rockies is not always associated with thunderstorms. The correlation of hail with thunderstorms has led to the use of the hail-thunderstorm ratio as an indicator of hailstorms (Reference 8). In a study done in Illinois by Changnon (Reference 8) nearly one-third of the thunderstorm days in the 245-day thunderstorm season did not have hail. Changnon has shown that in the long term average from 5 to 20 percent of the hail days in Illinois are not thunderstorm days. He also reports that the hail-thunderstorm ratios in Illinois vary from 3 to 7 percent, based on point averages.

He concludes, based on studies in three states, that the hail-thunderstorm ratio is strictly a function of the size of the area investigated and the density of observations within the area. He also did an intensive study of the differences in the hail-thunderstorm ratio within the state of Illinois. He attributes these to differences in meteorological conditions within the state. Thus, conditions leading to thunderstorms may vary in different locations, producing hail in some and not in others.

Therefore, although hailstorms and thunderstorms are closely associated, their connection will neither be further explored nor used in any attempt to model hail occurrence.

3. Climatological Differences.

Changnon has identified several different types of hail regions. These include marine effects, orographic effects, and effects due to macroscale synoptic weather conditions.

Hail production due to marine effects occurs where land-water contrasts, related to air-mass characteristics, produce sufficient atmospheric instability. The hail produced is usually small in size. The hail produced in two areas of the U.S. is primarily related to marine effects, the upper West Coast, and the area in the lee of the Great Lakes.

Orographic effects occur where there is thermodynamic lifting of air by mountains (the Rockies and Appalachian Plateaus) causing instabilities which lead to hailstorms. The effects of the Rockies are noticeable for 200 miles out.

The predominant cause of hailstorms in the remaining portion of the U.S. are synoptic scale atmospheric disturbances, usually related to frontal activity. The occurrence of hailstorms is also modulated by the existence of cities. Changnon (Reference 9) indicates that hailstorms are more frequent east of major cities in the midwest.

The significance of the causes of hailstorms lies in the amount and size of hail expected at a given location. Hail produced by marine effects is usually small and therefore, does not pose a threat to solar
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collectors. Hail produced by the other two causes can occur frequently in certain areas and be sufficiently large to be of concern. The area near the eastern edge of the Rockies, Colorado, Wyoming, Nebraska, and South Dakota, has experienced hail 4 or 5 inches in diameter. An area encompassing the eastern part of Nebraska, Kansas, Oklahoma, and the Texas panhandle experiences severe hailstorms caused by frontal activity which produce large hail. This area also experiences much thunderstorm and tornado activity due to the same cause.

Figure 2-1 shows a map of the U.S. with the three different causes of hailstorms indicated. This figure is adopted from Reference 10.


Although the life of a hail-producing storm may be as long as one or two hours, the duration at a point is much shorter, on the order of minutes. An entire storm consists of a series of convective cells, forming one after the other within the body of the main storm. Usually, the duration of hail fall at a point depends on the location of that point relative to the cell development and the state of cell development at the time.

Large hail is associated with a steep vertical temperature lapse rate. It appears that the strong vertical currents that produce large hail result from strong temperature gradients resulting from cooling of parcels of air by evaporation of portion of clouds and/or rain.

Two general factors are required for hail growth, a high liquid-water content, and cold temperature and long residence times aloft caused by high updraft speeds. Hail occurs when temperatures at ground level remain considerably above freezing; thus, it is a spring and summer phenomenon. Precipitation particles accumulate in the upper levels of the updraft. These particles which become hail embryos fall into a lower region where they grow. The embryos may become suspended in the updraft balanced by their fall velocity and the updraft velocity, or they may move upward or downward in the updraft. In any case they grow by accreting supercooled droplets carried toward the particles by the updraft. A hailstone will fall out if its fall velocity becomes greater than that of the updraft due to a decrease in the updraft or increase in the size of the hailstone. The stronger and more persistent the updraft, the larger the stones.

In general, the areal distribution of mean maximum temperature, mean noon dew-point temperatures, normal rainfall, and number of surface fronts, have an impact on hail patterns, causing local variations. Not all rain and thunderstorms produce ground level hail, either because stones produced aloft melt before reaching the ground or because they are not produced at all.
Figure 2-1. Hailstorm Regions Defined by Climatological Differences

M = MARINE
O = OROGRAPHIC
MS = MACROSCALE
5. Sources of Information.

A number of parameters associated with hailfall are significant. These include frequency of occurrence of a hailstorm at a point, duration, size of hailstones, areal density of hailstones, etc. The information available on hail is obtained in several ways.

There have been a number of hail experiments designed to test the effects of cloud seeding on the hail (References 11 and 12). In order to test the effectiveness of the seeding techniques, hail data were taken. These data provide information on such parameters as frequency of occurrence, hailstone size, areal density, etc.

Another source of information is the network of observers maintained by many agencies (References 9, 13, 14, 15, 16). The observers, which include farmers and other local people, are given cards on which they record pertinent hail parameters such as size. Figure 2-2 (taken from Reference 15) shows a typical card given to observers.

There are also accounts in Weather Bureau Records (References 1-7.) The Weather Bureau only records the occurrence of hail. However, they have published brief reports which include accounts of such information as hailstone size taken from accounts of casual observers.

Newspaper accounts discussing the hail occurrence and results from an observer's point of view, are a final source of data.

B. GEOGRAPHICAL AREAS AND HAIL.

Changnon (References 9 and 17) has identified a number of high-frequency hail areas. However, one must be careful in citing an area as a high hail area. Because of the climatic differences between areas, one high hail area may be a threat to solar arrays, while another is not. The midwest and central portion of the U.S. to the Rockies are considered to be high hail areas. There are many recorded occurrences of hailstones large enough and with sufficient areal density to cause major damage to solar panels over areas of tens of square miles. On the other hand, there are high hail areas on the West Coast which usually produce only small hail. The hail is damaging to fruit crops, such as pears, because small blemishes produced by the hail make the fruit unfit for sale. This type of hail would pose no threat at all to a solar array. Changnon identifies the following high-frequency hail areas in the central part of the U.S.:

(1) Wyoming - Cheyenne, Yellowstone, Casper, and Sheridan.

(2) Colorado - Denver, south east, and south central area.

(3) The Dakotas - Rapid City, South Dakota (violent hailstorms cited in Reference 5), other scattered areas and the boundary between the two states.

(4) New Mexico - north west of Las Vegas.
(1) **DATE of Storm**: ................. 1973

**Day of week**: .................

(2) **EXACT LOCATION** of observed hail occurrence: 

...... 1/4, S..., T..., R..., W of ....

(3) **HAIL** began ......... AM or ......... PM

**DAYLIGHT SAVING TIME**

HAIL lasted for ......... minutes.

RAIN lasted for ......... minutes and totalled ......... inches.

Did rain fall at same time as the hail: yes □; no □.

(4) During this time there were the following number of distinct HAIL BURSTS:

1 □ 2 □ 3 □ 4 □ 5 □ 6 □ more than 6 □ unknown □.

(5) **Size of LARGEST HAIL**: shot □; pea □; grape □; walnut □; golfball □; larger □.

Size of MOST COMMON HAIL: shot □; pea □; grape □; walnut □; golfball □; larger □.

(6) Average **SPACING of hailstones on the ground at end of storm** was ......... inches, or **DEPTH of hail** was ......... inches, or ground was JUST COVERED □.

(7) **HAIL** began: before rain □; at same time □; after rain began □; OR no rain □.

(8) **WINDS accompanying HAIL**: light □; moderate □; strong □; severe □.

(9) Was any hail SOFT or SLUSHY: yes □; no □: If yes, what percentage of stones were soft ......... %

(10) **SHAPE of largest stones**: conical □; flattened □; egg □; round □; other .........

**SURFACE STRUCTURE of largest stones**: smooth □; raspberry □; knobby □; other .........

(11) [Optional] Estimated **DAMAGE** at location above ......... %. Crop type .........

(12) Indicate on section grid above other nearby properties on which you know it hailed.

(13) Remarks:

Name .................................. Address ..................................

Check if you have a hail sample □ Home quarter location .... 1/4, S..., T..., R..., W of ....

Check if you require more cards □ Phone ................................ Exchange ................................

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**Figure 2-2. Typical Hail Reporting Card**
(5) Montana - central Montana and the area around Dillon.

(6) Central Texas, the area around Marfa, and the Panhandle.

(7) Upper Michigan.

(8) Indiana - north west area around South Bend.

(9) A 1300-mile long ridge beginning in south central New Mexico and extending uninterrupted across Oklahoma, Missouri, southern Illinois, southern Indiana, and into western Ohio.

(10) Illinois - southwestern, eastern and northwestern parts.

(11) Nebraska - most of Nebraska, including Platte River Valley and Omaha.

(12) Iowa - western, north central, and east central parts.

(13) Kansas - most of Kansas stretching into Kansas City, Missouri.

(14) Minnesota - extreme southwest and west central areas.

(15) Wisconsin - southwest, extending into east central Iowa and northwest Illinois.

These areas are shown in Figure 2-3, which gives the expected number of days with hail in 20 years according to Changnon (Reference 17). One of the difficulties with a map such as that shown in Figure 2-3 is that hail occurrence is recorded normally only where there are stations or observers of some type. For example, one form of hail information may be found in the records of crop losses due to hail. A low value in an uninhabited area may be due simply to lack of observations. On the other hand, solar arrays would most likely be deployed in inhabited areas, where information on the occurrence of hail is usually available. One exception is the sparsely populated southwest desert area, where one may assume that the occurrence of large hail is highly unlikely, even without substantiating records.

Although other areas of the country experience hail, some in large quantities, the major threat from larger hailstones (greater than 1 - 1.25 inches) lies in the central area shown in Figure 2-3. Figure 2-4 (taken from Reference 10) gives the average number of hail days for the part of the U.S. not covered in Figure 2-3.

As indicated above, Figures 2-3 and 2-4 give the average expected number of hail days. A high hail frequency does not necessarily mean high incidence of large hail. This point will be further developed in the next section.
Figure 2-3. Total Number of Days with Hail in an Average 20-Year Period
Figure 2-4. Average Number of Hail Days Per Year
A hailstorm probability model could contain a number of elements based on either empirical information, theoretical models or extrapolation from limited data. The model could also be based on one or more of the following parameters: frequency of occurrence of hailstorms, frequency of occurrence of hailstones of a given size; areal density of hailstones; physical parameters such as density, velocity, hardness, etc.; duration of the hailstorm; areal coverage by the hailstorm, and so forth.

The only information recorded in a systematic and continuous way is the point frequency of hailstorm occurrence. The other parameters cited above have been measured at small numbers of locations for short periods of time and in many instances, with the use of volunteer observers. In addition, much of the data gathered are not in a reduced form which could be applied to this problem. Furthermore, hailstorms are a very limited phenomenon. A hail area is considered from the standpoint of two different types of areas. First, hailstorm activity involves an envelope of semi-continuous hail areas called a hailswath. The average dimension found for storms in Illinois are 6 miles by 25 miles, with an average of two swaths per hail day about 20 miles apart. The following information was obtained in Alberta, Canada: for hailswaths considered there, 34 percent were 10-30 miles long, 31 percent were 31 - 50 miles long and 35 percent were more than 50 miles in length (Reference 9). If one takes into account the hailswaths measured in Alberta, one would expect hailswaths to cover areas from 150 mi² to 1000 mi².

The second type of hail area is the hailstreak which is a single volume of hail produced in a storm. Many hailstreaks form a major hailswath indicating that certain large storms produce several separate volumes of hail. The median size of a hailstreak found in Illinois was 7.9 mi² or the average dimensions were 1.1 miles by 5.9 miles (Reference 9).

Since a hailswath is a semi-continuous area of hail, the actual continuous area covered is better determined by considering the hailstreak.

According to Changnon (Reference 18) one sampling site per 2 mi² is needed to measure adequately the area of crop loss due to hail. Changnon then found (Reference 18) that a sampling density of one point per 0.25 mi², on the average detected twice as many days with damaging hail as did one point in 9 mi². This is due to the fact that for any given hailstorm hail damage is not widespread. In addition, any extrapolation of crop damage statistics to consideration of potential solar array damage is difficult because of greater susceptibility of crops to damage. Also, crop susceptibility varies with the time of year. In general, the damage threshold, in terms of hailstone size, will be significantly higher for solar arrays than for crops.
A. NUMBER OF HAILSTORMS PER YEAR.

The availability of information on the number of hailstorms per year is covered in Section II.A.5. Figures 2-3 and 2-4 provide values as a function of geographical location. The values provided in these figures are used to arrive at the results presented later in the report.

The number of hailstorms per year is usually given as average annual number of hail days. This is the number of days that hail was reported by a given observation station or observer. Thus each hail day is assumed to be one occurrence of hail. The number of hail days is used here as a point value.

B. NUMBER OF EXPECTED HAILSTONES OF A GIVEN DIAMETER.

The information available on hailstone size distribution is sparse. Several studies have been performed which obtained information on hailstone size, but only represented a limited area of the country. In addition, numerous activities have involved gathering hailstone data (References 9, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26). Most of the data are based on reports of volunteer observers. The accuracy of such data may be open to question. However, that is not the major problem in attempting to derive a hailstone size distribution as a function of geographical location. The main problem is the lack of data for most areas. The data taken to date have been confined mainly to three general areas, Illinois, Colorado, (near Denver), and Alberta, with some data taken in the Dakotas, Arizona, Oklahoma, and New England.

Therefore, to develop even a cursory geographic distribution for hailstone size, broad generalizations will have to be made. Based on a discussion with Changnon and use of Reference 10, the Continental U.S. has been divided into three areas (see Figure 3-1). The available data from each region were plotted and the envelopes drawn which encompassed all of the available data (see Figures 3-2, 3-3, and 3-4). No envelope was constructed for Region III because only one data set is available. The graphs are cumulative probability plots, giving the probability of obtaining hailstones of a given diameter or greater, given that a hailstorm has occurred.

Most of the data available are for hailstone diameters of 1 inch or less. In order to obtain probabilities for larger diameters, an extrapolation has to be made. The plots in Figures 3-2, 3-3, and 3-4 are log-log plots. Two envelope curves were plotted so that the slope matched that of the plotted area. An upper limit and a lower limit envelope were plotted in order to bound the hailstone size data. The slopes of the plotted data are continuously varying. Thus, it can be seen from the figures that a simple extension of the curve for diameters of 1 inch or less, at the last slope recorded would give a curve outside of the upper envelope curve. Therefore, the envelope curves are drawn with a slope much steeper than the slope of the curves which represent smaller sizes. The basis for doing this is simply engineering judgement derived from discussions with personnel working with hail data. It was felt to be unreasonable to plot
Figure 3-1. Hail Regions
Figure 3-2. Probability of Obtaining Hailstones of Diameter Equal to or Greater than D, (Data from Illinois)
Figure 3-3. Probability of Obtaining Hailstones of Diameter Equal to or Greater than D, (Data from Colorado, Alberta, and Oklahoma)
NOTE: Data used to develop this graph were taken from Reference 20. Results of this graph were used in Region III.

Figure 3-4. Probability of Obtaining Hailstones of Diameter Equal to or Greater than D, (Data from Arizona)
cumulative probability curves which have a constant slope from smaller sizes to the larger sizes.

One may question why the cumulative probability curves are extended to larger sizes, if so little data are available for these larger sizes. As cited in the introduction, there have been sufficient reports of large size hailstones so that their existence is a certainty. Therefore, except in Region III, in Figure 3-1, they cannot be ruled out. Therefore, the probability distribution provided for large stones is the "best guess" at this point. A range is provided by the envelopes. An average value would be located in the central region of the area bounded by the envelopes.

Table 3-1 gives the results for the three regions shown in Figure 3-1 which are taken from the plots in Figures 3-2 to 3-4. The values given in Table 3-1 are the probability of getting hailstones of a given diameter or larger. These were obtained from the envelopes of the plots shown in the figures. The plots shown were constructed from data taken from the references indicated. The data provided in the references were for different size intervals and were a conglomeration of data obtained from hailpads and data obtained from observers. Changnon (Reference 14) found that for hailstones larger than 0.5 inch the size distribution derived from observers agreed well with those derived from hailpad data. In order to account for uncertainties in data acquisition, especially when taken from observer reports, a data interval about a given size was used. For example, in obtaining the plots, the value used for the probability for diameters greater than 1 inch may have included reports for diameters from 0.8 inch to 1.25 inches. In some cases, data were reported for diameters greater than 1.0 inch, greater than 0.75 inch, etc., and due to the nature of data, the information was used exactly as reported. In other words, a judgement was made on what data to include in given size intervals. The closeness of the curves suggests that this was a reasonable approach to take.

The cumulative probability distribution for Region III has no sizes greater than 1 inch. There are very few reports of sizes greater than 1 inch in this area. The data used for this region were taken from Reference 20 and represent the hail environment on top of a mountain peak 9184 ft (2800 m) above sea level. Therefore, this environment represents an upper limit for Region III and is probably too severe for lower elevations. An important question is whether hail of a significant size occurs in the flat areas of the southwest with an abundance of sunshine. Discussions with personnel at the University of Arizona at Tucson, the National Weather Service at Tucson, and the Atmospheric Science Lab at White Sands Missile Range (Reference 27) indicate that hail above 0.75 inch is not usually observed in the desert areas. They report that they have not experienced large hail or heard of damage due to hail. There is also no record of damage due to hail in these areas.

Therefore, the hail size distribution given in Figure 3-4 and Table 3-1 should be applied only to the mountainous areas of Region III. Except for the extreme northern end of Region III, any threat from hail to solar panels should be discounted. In the northern area there are reports of frequent hail but it is usually small. Therefore, under normal
Table 3-1. Cumulative Probability of Obtaining Hailstones of a Given Diameter or Greater*

<table>
<thead>
<tr>
<th>Diameter (inches)</th>
<th>Region I</th>
<th>Region II</th>
<th>Region III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper</td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td></td>
<td>Limit</td>
<td>Limit</td>
<td>Limit</td>
</tr>
<tr>
<td>≥0.25</td>
<td>0.94</td>
<td>0.50</td>
<td>1.0</td>
</tr>
<tr>
<td>≥0.50</td>
<td>0.75</td>
<td>0.20</td>
<td>0.96</td>
</tr>
<tr>
<td>≥0.75</td>
<td>0.46</td>
<td>0.085</td>
<td>0.65</td>
</tr>
<tr>
<td>≤1.00</td>
<td>0.26</td>
<td>0.030</td>
<td>0.40</td>
</tr>
<tr>
<td>≥1.25</td>
<td>0.15</td>
<td>0.006</td>
<td>0.25</td>
</tr>
<tr>
<td>≥1.50</td>
<td>0.07</td>
<td>0.0012</td>
<td>0.16</td>
</tr>
<tr>
<td>≥2.00</td>
<td>0.008</td>
<td>0.00011</td>
<td>0.03</td>
</tr>
<tr>
<td>≥3.00</td>
<td>0.00025</td>
<td>4 x 10^-6</td>
<td>0.0013</td>
</tr>
<tr>
<td>≥4.00</td>
<td>0.00002</td>
<td>3.5 x 10^-7</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table 1 is obtained from the envelope curves in Figures 6-8 and is to be used in conjunction with Figure 5.

NOTE: Data are based on hailpad and observer data.

*Given that a hailstorm is occurring.
circumstances it should pose no problem. It is possible, however, that some uninhabited areas have received larger hailstones.

C. AREAL DENSITY OF HAILSTONES.

Data on areal density of hailstones are generally not available. Most of the networks used heretofore for gathering data were dependent on volunteer observers. They were asked to record such information as stone size and storm duration, but there are few reports of areal density. One of the few, which is used here, is available in Reference 12. A table is provided in this report of the maximum number of stones of a given diameter and the average number per square foot, based on a 6-year study. These data, from Reference 12, were taken in Illinois and are summarized in Table 3-2.

The data were taken with hailpads. Since hailpads are usually one ft² in size and spacing of the hailpads in this study ranged from 3 miles apart to 275 feet, the areal density of only the smaller stones can be reasonably measured. Large stones are spaced far enough apart, so that areal density for them cannot really be measured in this way because of the small size of pad. The occurrence of the larger stones is poorly detected by the spacings used because of the small area over which they occur.

In order to obtain estimates of areal density for the larger stones, the cumulative fractional number of the total was plotted on log-log paper (see Figure 3-5). This value was determined by taking the ratio of the number of stones per ft² (of a given diameter or greater) to the total number per ft² (all diameters). The number of stones per ft² was taken from Table 2. The ratio was then plotted versus hailstone diameter and the curve extrapolated to obtain the values for stones of diameters greater than 1 inch. This was done for both the average and the maximum number of stones per ft². Values of this ratio for large stones were taken from the curve and converted to number per ft². These are given in Table 3-3.

Since Table 3-3 represents the only data available it was used in the study discussed here. The values given in Table 3-3 represent two distinct hail regimes. One is the average areal density expected on the basis of six years of data, which is a much milder environment than the maximum areal density. The recommendation here is to use the average density in Regions I and III (Figure 3-1) as the representative of the hail environment expected there and the maximum density in Region II. One basis for dividing the country into these regions is observed hail damage. Hail damage is most intense in Region II, which therefore must be subject to more severe hailstorms. The maximum density distribution seems to conform more to the reports associated with instances of large hail fall (References 1-7) with the occurrence of widespread damage. For example, in the case of 2-inch hailstones, the maximum distribution gives one stone per 2 ft² and the average distribution gives one stone per 50 ft². The incidents of widespread damage were most probably caused by hail whose areal density was closer to the former than the latter number.
Table 3-2. Number of Hailstones Per ft² (Per Hailfall, Data Taken in Illinois)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>1/8</th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
<th>1</th>
<th>&gt;1</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967-1968</td>
<td>100</td>
<td>14</td>
<td>12</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>129</td>
</tr>
<tr>
<td>1971-1972</td>
<td>79</td>
<td>11</td>
<td>7</td>
<td>0.9</td>
<td>0.1</td>
<td>0.04</td>
<td>98</td>
</tr>
<tr>
<td>1973-1974</td>
<td>105</td>
<td>14</td>
<td>4</td>
<td>0.4</td>
<td>0.1</td>
<td>0.01</td>
<td>123</td>
</tr>
<tr>
<td>Average</td>
<td>94.6</td>
<td>13.0</td>
<td>7.7</td>
<td>1.1</td>
<td>0.4</td>
<td>0.02</td>
<td>117</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time Period</th>
<th>1/8</th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
<th>1</th>
<th>&gt;1</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971-1972</td>
<td>1146</td>
<td>186</td>
<td>215</td>
<td>56</td>
<td>17</td>
<td>7</td>
<td>1627</td>
</tr>
<tr>
<td>1973-1974</td>
<td>1454</td>
<td>251</td>
<td>131</td>
<td>34</td>
<td>5</td>
<td>2</td>
<td>1877</td>
</tr>
<tr>
<td>Average</td>
<td>1280.0</td>
<td>213.0</td>
<td>201.3</td>
<td>66.0</td>
<td>15.7</td>
<td>6.7</td>
<td>1783</td>
</tr>
</tbody>
</table>
Figure 3-5. Fraction of Total Hailstones Per Square Foot for Hailstones of a Given Diameter or Larger

NOTE: Fraction of maximum based on 1,783 hailstones/ft$^2$ total.
Fraction of average based on 117 hailstones/ft$^2$ total.
Table 3-3. Number of Hailstones Per Hailfall of a Given Diameter or Greater Per ft²

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>9.22</td>
<td>290.</td>
</tr>
<tr>
<td>0.75</td>
<td>1.52</td>
<td>88.</td>
</tr>
<tr>
<td>1.0</td>
<td>0.45</td>
<td>22.</td>
</tr>
<tr>
<td>1.5</td>
<td>0.064</td>
<td>2.1</td>
</tr>
<tr>
<td>2.0</td>
<td>0.019</td>
<td>0.45</td>
</tr>
<tr>
<td>3.0</td>
<td>0.003</td>
<td>0.05</td>
</tr>
<tr>
<td>4.0</td>
<td>0.0007</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Secondly, as cited above, areal density values taken on one ft² pads separated by up to 3 miles could easily be underestimated or overestimated. Therefore, damage reports were given considerable weight in selecting the maximum areal density for use in Region II.

One final note, the use of the areal density distribution from Table 3-3 for Region III requires truncation for hailstones greater than one inch in order to conform with Table 3-2.

D. DURATION OF HAILSTORMS.

This parameter does not have much importance in the approach discussed in this report. The approach discussed makes use of parameters whose value has been integrated over the duration of a hailstorm. Therefore, the hailstone duration does not enter the calculations directly. Changnon (Reference 9) cites averages of 10 minutes for hailstorms in Alberta (characteristic of area east of the Rockies into the Great Plains) and averages of 6.5 to 9.5 minutes in Illinois. Therefore, average duration of 5 to 10 minutes are to be expected with highs of up to 45 minutes.

E. PHYSICAL PARAMETERS OF HAIL.

The physical hail parameters include size (previously discussed), density, velocity, and mass as well as quantities derived from these, such as kinetic energy and momentum.

Gringorten (Reference 29), as do most other authors, uses a density for hail of 56 lb/ft³ (900 kg/m³). Gringorten discusses the terminal velocity of hailstones, using the formula,
where

\[ W = K \sqrt{d} \]

\( W = \) terminal velocity in m/s  
\( d = \) diameter in centimeters  

\( K = 11.5 \)

Gringorten gives \( K \) as function of atmospheric pressure, in order to account for changes in location:

<table>
<thead>
<tr>
<th>Atmospheric Pressure, millibars (mb)</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>15.9</td>
</tr>
<tr>
<td>850</td>
<td>16.3</td>
</tr>
<tr>
<td>700</td>
<td>17.5</td>
</tr>
<tr>
<td>500</td>
<td>20.0</td>
</tr>
<tr>
<td>400</td>
<td>22.0</td>
</tr>
</tbody>
</table>

Using the value of \( K \) for 5000 feet above sea level, 850 mb, which is 16.3, Gringorten obtained the following values of terminal velocity:

<table>
<thead>
<tr>
<th>Diameter, ( d ) cm</th>
<th>Terminal velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.6</td>
<td>42</td>
</tr>
<tr>
<td>7.9</td>
<td>46</td>
</tr>
<tr>
<td>8.9</td>
<td>49</td>
</tr>
<tr>
<td>10.2</td>
<td>52</td>
</tr>
</tbody>
</table>

Friedman (Reference 29) gives the following values for terminal velocity:

<table>
<thead>
<tr>
<th>Diameter, ( d ) inches</th>
<th>Terminal velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>15.2</td>
</tr>
<tr>
<td>2.5</td>
<td>21.9</td>
</tr>
<tr>
<td>3.8</td>
<td>27.4</td>
</tr>
<tr>
<td>5.1</td>
<td>32.0</td>
</tr>
<tr>
<td>6.4</td>
<td>36.0</td>
</tr>
<tr>
<td>7.6</td>
<td>39.6</td>
</tr>
<tr>
<td>8.9</td>
<td>42.7</td>
</tr>
<tr>
<td>10.2</td>
<td>45.4</td>
</tr>
</tbody>
</table>

Foster and Bates (Reference 30) compute terminal velocity using the following equation:

\[ W = \sqrt{\frac{2 \rho_h g}{3 \frac{1}{C_d} \rho_a}} \cdot \sqrt{d} \]

where
\[ \rho_h = \text{hailstone density} \]

\[ g = \text{acceleration of gravity} \]

\[ \rho_a = \text{density of air} \]

\[ C_D = \text{drag coefficient} \]

The kinetic energy of a hailstone is given by \( \frac{1}{2} m V^2 \) where \( V \) is the hailstone velocity \((V = V_T + V_W)\), \( V_T \) is the terminal velocity, \( V_W \) is the wind-driven velocity, and \( m \) is the hailstone mass.

Friedman (Reference 29) gives the following values of kinetic energy (with no wind):

<table>
<thead>
<tr>
<th>Diameter, ( d ) cm</th>
<th>Kinetic Energy (KE) joules</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>2.3</td>
</tr>
<tr>
<td>3.8</td>
<td>11.7</td>
</tr>
<tr>
<td>5.1</td>
<td>39.3</td>
</tr>
<tr>
<td>6.4</td>
<td>98.3</td>
</tr>
<tr>
<td>7.6</td>
<td>217.9</td>
</tr>
<tr>
<td>8.9</td>
<td>503.7</td>
</tr>
<tr>
<td>10.2</td>
<td>918.4</td>
</tr>
<tr>
<td>11.4</td>
<td>1567.3</td>
</tr>
<tr>
<td>12.7</td>
<td>2585.8</td>
</tr>
</tbody>
</table>

The hailstone momentum is given by \( mV \).

Using the terminal velocities provided by Friedman the following momenta are obtained:

<table>
<thead>
<tr>
<th>Diameter, ( d ) cm</th>
<th>Momentum kg-m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>1.47x10^-2</td>
</tr>
<tr>
<td>2.5</td>
<td>1.69x10^-1</td>
</tr>
<tr>
<td>3.8</td>
<td>7.14x10^-1</td>
</tr>
<tr>
<td>5.1</td>
<td>1.98</td>
</tr>
<tr>
<td>6.4</td>
<td>4.39</td>
</tr>
<tr>
<td>7.6</td>
<td>8.26</td>
</tr>
<tr>
<td>8.9</td>
<td>13.5</td>
</tr>
<tr>
<td>10.2</td>
<td>22.4</td>
</tr>
</tbody>
</table>
A. PROBABILITY MODEL.

A probability model was developed for use in estimating the probability of impact by hailstones of a given size over a given period of time. The model is divided into several elements and makes use of three types of information about hailstones: the average annual number of hail days; the expected frequency distribution of hailstone sizes, given that a hailstorm has occurred; and the areal density of hailstones. These three types of hail information are discussed in Section III, paragraphs A, B, and C respectively. The values provided there are used in the probability model discussed here.

The first element of the model is the probability of obtaining a given number of hailstorms in a year. In this case the probability of occurrence of n hailstorms in a year is assumed to be given by a Poisson distribution. Gringorten uses this type of distribution (Reference 28) in his report. A discussion of the use of this distribution to treat hail events appears in References 31 and 32. The principal criterion for applying the Poisson distribution to an event, such as a hailstorm, is that the events be both comparatively rare and independent. In general, the mean number of days with hail (mean hail frequency) is small compared to the number of days in a year which comprise the hail season. Secondly, it is assumed here that no two storms which occur are dependent. This assumption may break down in areas with a large annual frequency of storm days. For instance, in the summer a series of storms associated with the passage of a single storm front may lead to hail occurrences which are not independent.

The Poisson distribution has a probability function given by the following:

\[ f(n) = e^{-\lambda} \frac{\lambda^n}{n!} \]  

(1)

where \( n \) is the number of hail days in years \((f(n))\) gives the probability of obtaining \( n \) hail days), and \( \lambda \) is the mean annual number of hail days.

It is often convenient to use what is called the distribution function given by:

\[ F(N) = \sum_{n=0}^{N} f(n) \]  

(2)

where \( F(N) \) is the probability of obtaining \( N \) hail days or less in a year.

Thom (Reference 31) discusses the use of the negative binomial distribution for those cases which are not adequately described by a Poisson distribution.
One form of the negative binomial distribution is given by the following equation:

\[ f(n) = \frac{\Gamma(K+n)}{\Gamma(n+1) \Gamma(k)} \frac{p^k}{(1+p)^{k+n}} \]  

(3)

where \( \Gamma \) represents the gamma function and \( k \) and \( p \) are parameters of the distribution.

Values of \( p \) and \( k \) can be determined from the following:

\[ p = \frac{S^2 - \lambda}{\lambda} \]

\[ k = \frac{\lambda^2}{S^2 - \lambda} \]

where \( S^2 \) is the sample variance.

The reader is referred to Reference 31 for a more detailed discussion of the negative binomial distribution.

Changnon and Schickendenz (References 32 and 33) applied both distributions to hail occurrence data from Illinois and found a tendency for summer data to be fitted by the negative binomial distribution and the annual data to be fitted by the Poisson. This occurs because the summer data are more likely to be a series of dependent events (such as a series of storms), and hence, better represented by the negative binomial distribution which allows for dependence.

Based on these results and the greater simplicity of the Poisson distribution, it was used in the current study to determine the probability of obtaining \( N \) or less hail occurrences in a year. However, it must be emphasized that implicit in its use is the assumption of independence of events which may not always hold.

Use of Equations 1 and 2 gives the probability of obtaining \( N \) or less occurrences of hail in a given year:

\[ \sum_{n=0}^{N} e^{-\lambda} \frac{\lambda^n}{n!} \]

In order to find the probability of obtaining hailstones of diameter \( d \) or larger the size frequency distribution given in Table 3-1 in Section III.3 is used. In this case, since the parameter of interest is the probability of obtaining hailstones of a given size one or more times, the probability of obtaining hailstones of diameter less than \( d \) in each of \( N \) occurrences is determined. If the cumulative probabilities found in Table 3-1 are subtracted from 1, the probability of obtaining hailstones of diameter less than \( d \), \( Q(d) \), is found. The value of \( N \) is selected sufficiently
large so that additional terms which are omitted contribute insignificantly to the total probability; for this case \( N = 20 \) is chosen. Therefore, the probability of only obtaining hailstones of diameter less than \( d \) is given by:

\[
\sum_{n=0}^{20} \frac{e^{-\lambda} \lambda^n}{n!} Q^n(d) \quad (4)
\]

It follows that the probability of obtaining only hailstones less than diameter \( d \) in \( K \) years is:

\[
\left[ \sum_{n=0}^{20} \frac{e^{-\lambda} \lambda^n}{n!} Q^n(d) \right]^K
\]

Therefore, the probability of obtaining hailstones of diameter \( d \) or greater in \( K \) years, \( P(d) \) is given by:

\[
P(d) = 1 - \left[ \sum_{n=0}^{20} \frac{e^{-\lambda} \lambda^n}{n!} Q^n(d) \right]^K \quad (5)
\]

The value of \( P(d) \) provides the probability of obtaining hailstones of a given diameter or larger over a period of time. However, this does not determine the actual probability of a hit in an area of a given size. In order to do so, the areal density of hailstones is required. The probability of being hit by a hailstone is given by a Poisson distribution. If the expected areal density of hailstones of a given diameter is \( M(d) \) (number of stones per unit area) the average number of hits on an area \( A \) is:

\[
A M(d)
\]

The probability of getting \( n \) hits on an area \( A \) is given by:

\[
e^{-A M(d)} \frac{[A M(d)]^n}{n!} \quad (6)
\]

The probability of getting at least one hit is given by:

\[
1 - e^{-A M(d)} \quad (7)
\]

based on 1 minus the probability of no hits.

The values of \( M(d) \) used here are taken from Table 3-3. The values under the average areal density are used to represent Regions I and III, and under the maximum areal density are used to represent Region II.

Therefore, the probability of a given fractional area, \( A \), of a module being hit once or more by hailstones of diameter greater than or equal to \( d \) in \( K \) years is:
\[
P(A,d) = \left[1 - \sum_{n=0}^{20} \frac{e^{-\lambda} \lambda^n}{n!} Q^n(d) \right]^K \left(1 - e^{-A M(d)} \right) \tag{8}
\]

Assume that a module consists of a number of areas, \(A_i\), each of which is susceptible to damage from hits by hailstones of diameter \(d_i\) or larger. For instance, a cell may be subject to damage from direct hits by hailstones smaller than those which would cause damage if they struck a point over the substrate. Thus, a module of a given total area, \(A\), can be divided into fractional areas \(A_i\), each of which is susceptible to hailstones of a different size.

The risk of damage to a module from hail is dependent on the combined probability of each separate area being bit by hail of the size to which it is susceptible. This probability is given by:

\[
P(A) = 1 - \prod_{i=1}^{\eta} \left[1 - P(A_i,d_i)\right] \tag{9}
\]

where \(\eta = \text{Total number of areas.}\)

The mean time between hits, MTBH, is given by:

\[
MTBH = -\frac{T}{\ln \left[1 - P(A)\right]} \tag{10}
\]

where \(T\) is the time of observation and is equal to \(K\) in this case. This equation is adopted from Reference 34 and derived from the concept of mean time between failure. In this case, the MTBH is an indication of time between hits by hailstones which could be damaging to a collector. For example, a MTBH of five years means that the mean time between successive hits of a collector by hailstones of a given size is five years.

B. RESULTS OF APPLYING THE PROBABILITY MODEL TO HAIL DATA.

A sensitivity analysis was performed using the hailstone model given by Equations 8, 9, and 10 by varying the values of the probability of obtaining hailstones, Table 3-1, and the areal density of the hailstones. The quantity determined was the mean time between hits for a 20-year exposure for a panel 4 feet by 4 feet. The analysis was performed for one, three, five, and nine hailstorms per year, and hailstone sizes of 1, 1.5, 2, and 3 inches.

The results of the analysis are given in Figures 4-1 through 4-5. In these figures, MTBH is plotted versus the probability of obtaining hailstones (given that a hailstorm has occurred). A separate curve is plotted for each of a given number of areal densities.

Areas of each set of curves were sectioned off, based on the probability of occurrence and areal density ranges, given in Tables 3-1 and 3-3, respectively, for a given size hailstone. For the purpose of this analysis, the entire 4 feet x 4 feet panel is assumed susceptible to damage from the same size hailstone. This simplification is required.
Figure 4-1. Region I: MTBH vs Probability of Occurrence of Hailstones of a Given Size for an Area with One Hailstorm/Year Assuming a 20-Year Exposure
Figure 4-2. Region I: MTBH vs Probability of Occurrence of Hailstones of a Given Size for an Area with Three Hailstorms/Year Assuming a 20-Year Exposure
Figure 4-3. Region II: MTBH vs Probability of Occurrence of Hailstones of a Given Size for an Area with Three Hailstorms/Year Assuming a 20-Year Exposure
Figure 4-4. Region II: MTBH vs Probability of Occurrence of Hailstones of a Given Size for an Area with Five Hailstorms/Year Assuming a 20-Year Exposure
Figure 4-5. Region II: MTBH vs Probability of Occurrence of Hailstones of a Given Size for an Area with Nine Hailstorms/Year Assuming a 20-Year Exposure
because of the large number of combinations of probability of occurrence and areal density which was used. This should not affect the results by an appreciable amount. The ranges result from use of the upper and lower values for probability of occurrence and the average and maximum values for areal density. The range of values of MTBH for each hailstone size denotes the range of uncertainty for MTBH in the given geographical region. A value was also selected from the center of the range as the most probable value. The range of values for a given size of hailstone, is given for Regions I and II with the most probable value. Region III is represented by a single point since only one set of probability of occurrence and areal density values was considered appropriate.

The analysis of Region I is based on use of the curves for one and three hailstorms per year, and that Region II on three, five and nine hailstorms per year. Region III is based on one, three and five hailstorms per year, the latter value representing mainly mountainous areas in the northern part.

The selection of annual number (range) of hailstorms for this phase of the analysis is based on the number of storms most prevalent for the region in question.

The sensitivity analysis indicates the MTBH is sensitive to the probability of occurrence and areal density in some range of values and insensitive in others. For example in the case of three annual hail days, the curves show that the MTBH is insensitive to probability of occurrence for values above 0.1 for areal densities of less than one hailstone per square foot. Probability of occurrence values above 0.1 mainly include hailstones of 1-inch or less diameters. In terms of areal density the MTBH becomes insensitive to the parameter for values above 2 per square foot for all probabilities and down to 0.5 per square foot for probabilities below 0.1. The regions of insensitivity varies for curves representing different numbers of hail days. Use of this information is helpful in determining which parameters need to be considered further for possible improvement in accuracy.

The results of this phase of the analysis are summarized in Table 4-1 giving the ranges and most probable values. The MTBH are given for the appropriate region based on the annual number of hail days for selected hailstone diameters. A range of values and an average or most probable value obtained by visual inspection of Figures 4-1 through 4-5 are given. The following summary is obtained by considering Table 4-1. In Region I, 1-inch hailstones are responsible for MTBH less than 20 years; while 2-inch hailstones lead to MTBH greater than 20 years. In Region II, the MTBH for 1-inch hailstones is very short, while the MTBH for 2-inch hailstones spans a very large range. The lower end of the range presents MTBH of less than 20 years; however, the upper end presents MTBH well in excess of 20 years with the average larger than 20 years.

The MTBH values provided in Figures 4-1 through 4-5 and Table 4-1 may appear high at first glance. However, these numbers are provided based on data obtained at point locations. Two factors must be taken into account. First, the assumption is made that the size distribution from one point location can be transferred to a large area. This has inherent
Table 4-1. Mean Time Between Hits for Regions I, II, and III

<table>
<thead>
<tr>
<th>ANNUAL NUMBER OF HAIL DAYS</th>
<th>HAILSTONE SIZE (inches)</th>
<th>REGION I MTBH (years)</th>
<th>REGION II MTBH (years)</th>
<th>REGION III MTBH (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>RANGE</td>
<td>AVERAGE</td>
<td>RANGE</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>4 - 30</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>15 - &gt;1000</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>130 - 21000</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.2 - 11</td>
<td>4</td>
<td>0 - 2.8</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>5 - 460</td>
<td>50</td>
<td>2 - 42</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>40 - 21000</td>
<td>250</td>
<td>11 - 750</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>500 - 21000</td>
<td>21000</td>
<td>500 - 21000</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0 - 2.8</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.1 - 30</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.8 - 420</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>320 - 21000</td>
<td>21000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0 - 2.8</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>0 - 25</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.6 - 240</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>170 - 21000</td>
<td>21000</td>
<td></td>
</tr>
</tbody>
</table>
difficulties in that the hail environment may be quite different in going from one point to another. Furthermore, the distribution function of two-inch and larger hailstones was obtained by extrapolation of data for smaller hailstones. Therefore, the results presented represent, at best, an approximation. The second factor is the rather long time periods involved, 20 or 30 years. Therefore, although hailstones of a given size may not be frequent, they may have a surprisingly high probability of occurring at least once in 20 to 30 years.
CONCLUSIONS AND RECOMMENDATIONS

This report presents the data and approach used in a study to develop a hail environment for use with solar photovoltaic modules. The results are given in terms of the mean time between hits for a given region. The difficulty with taking a rather small amount of data and generalizing it to a large area is summed up by Changnon (Reference 9, p. 626):

"Hail, whether it is viewed as the quickly melting hailstones on a patio, the hailstorms during one June week in Colorado, or the hail season in Alberta, exhibits enormous variability that exceeds that of most other weather conditions. This time and space variability is the key characteristic of hail, and the variability results because hail falls are such small scale areal phenomena and relatively infrequent events at any one point."

Changnon commented further on the time variability (Reference 35, p. 211):

"The temporal variation of these events is sufficiently great that accurate data for any particular 10- and 20-year period may provide a point average that is considerably above or below the time longterm average for that point."

This point is also illustrated by hail information provided by Changnon (Reference 36) for the area around St. Louis, Missouri. Figure 5-1 shows the total number of hail occurrences (hail days) from 1971 to 1975 for an area with a radius of 11 miles (17.5 km). There is greater variability in this area which is 380 mi$^2$ (9.6 x 10$^8$ m$^2$) than the areas considered in the current study, which are much larger. For most areas, the total number of occurrences is within the range given for the St. Louis area in Figure 2-3. According to Figure 2-3, the St. Louis area had an average of 20-50 hail days in 20 years or 5 to 12.5 hail days in 5 years. There are areas shown in Figure 5-1 where the number of occurrences is less than 5. Thus, a generalized map, such as that shown in Figure 2-3, can never account for local variations. Therefore, the maps such as the one in Figure 3 may be overly conservative for a given point in the region considered, but on the other hand may underestimate values for some other point.

Figure 5-2 gives a map showing the maximum hailstone diameter observed for the St. Louis area for 1971 to 1975. In other words, these are the largest hailstones observed at a given point in that time period. The maximum diameter shown for this 5-year period is 2 inches. As in the case of Figure 5-1, there is great variability over this region in maximum stone size, ranging from less than 1/8 inch to over 2 inches.

Figure 5-3 shows the variability in the maximum number of stones per ft$^2$ received during any storm in the 5-year period. The same variable pattern is shown in this case also.
Effect denotes effect of city on hailstorm occurrence.

Figure 5-1. Total Hail Occurrences, 1971-1975
Figure 5-2. Maximum Hailstone Diameters, 1971-1975
Figure 5-3. Maximum Number of Stones (per Square Foot) 1971-1975
One can conclude that the results from a model such as that used here will overestimate the potential damage from hail in some areas and underestimate it in other areas.

Two aspects of the problem need further consideration. First, there are data available which have not been reduced. Reduction of these data would provide more information from which to determine size distribution for areas in which hail damage is a serious problem. The second aspect, and more important to consider, is interpretation of the data that are reduced and nonreduced. This includes both data directly interpretable in terms of hail parameters and data which are indirectly interpretable. The first type consists of such things as hailstorm observations and hailstone size data; the second type consists of such things as observations of radar echoes. The ideal approach would be to correlate the first type of data where it exists, with the second. This is important since the first type exists for few locations, while the second type is available for more locations.

It is recommended that a study be initiated to cover those areas where hail presents a serious problem and where photovoltaic applications have a significant potential. The study should be conducted by personnel familiar with hail statistics and meteorological phenomenon. The special characteristic that is required is the ability to assess the data available and relate it to various meteorological and climatic conditions. In this way, hail models may be developed for specific locations affected by a given type of climatology. Also, meteorological observations which have been recorded can be statistically analyzed in order to develop a predictive model whose end objective is to provide an estimate of a given set of hail parameters for an area.

Therefore, any further studies performed to assign hail risk should be performed for a specific location, taking into account available records for hail and other meteorological parameters and prevailing meteorological, topographic and man-made influences. Because of the immense effort involved in correlating and refining dissimilar, nationwide data, future studies should concentrate on testing specific areas where large surfaces of solar arrays will be exposed to a significant risk.

Finally, an alternate approach to the use of very limited available data could be taken. Such an approach would be a statistical sampling of people in hail prone areas to determine the largest hailstones and areal density associated with it that occurred within their memory. Using the frequency of occurrences of severe events and the length of the experience of people involved, a recurrence frequency for given hailstone events can be determined.
REFERENCES


27. Private communications with personnel from University of Arizona at Tucson, National Weather Service at Tucson, and Atmospheric Science Lab at White Sands Missile Range.


BIBLIOGRAPHY


